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The Impact of Discourse on Math Learning in Upper Elementary

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Abstract

Upper-elementary mathematics becomes increasingly complex, and the gap between fluency and ineptitude grows. Considering the importance of math competency, the educator must act to narrow this achievement gap. This six-week action research study examined the effect of the implementation of teaching and encouraging student application of differentiated discourse strategies on mathematical achievement and empowerment on twenty-two nine-to-twelve-year-old suburban students. Qualitative and quantitative data analysis yielded three key themes: nominal growth in student achievement, a marked increase in mathematical modeling, and a considerable shift in perception of discourse responsibility, impacting student mindset, behavior, and participation. Findings suggest that student engagement in mathematical discourse is a transformative practice. Further research is required to quantify academic gains over an extended period of intervention and the influence of adult execution in identifying the upper and lower zones of proximal development.

Keywords: discourse, mathematics, constructivism, empowerment, differentiation, engagement, self-efficacy, Montessori

The Impact of Discourse on Math Learning in Upper Elementary

Math competency is fundamental to success in life and higher education (Bloom, 2009). Math competency directly contributes to career opportunities, but it also enhances everyday understanding of stocks, sports scores, medical information, election data, insurance, and law (Paulos, 1988). All schools have math standards in place to guide instruction. For example, in the United States, Common Core Standards set expectations for instruction and student outcomes in public schools. Math in upper elementary grades becomes increasingly complex and abstract, requiring a stronger emphasis on problem-solving by using prerequisite skills to solve problems that require students to go beyond manipulatives or models to a deeper level of conceptual understanding (Bender, 2013). Upper elementary includes fourth through sixth grade, comprising children ages nine to twelve years old. Computing correct answers are not the emphasis; instead, students must combine math content with developed habits of mind to solve increasingly complex problems.

The complexity of math work in upper elementary requires a greater emphasis on critical thinking, which contributes to widening the gap between those who understand mathematical concepts and problem solving and those who need support or remediation (Bloom, 2009). To ensure that students attain mathematics proficiency, the educator should reflect on teaching methods (Bloom, 2009) and seek to transform practices and problem resolution in the classroom.

Students in the same classroom, receiving the same instruction, vary broadly in their mastery of grade-level math content. Variations in student interactions, engagement, success during lessons, and formative and summative assessments, are all evidence that educators face a challenge. Presuming all students have access to grade-level content, strategies must be in place to close achievement gaps and meet growth goals to attain competency and maximum growth in mathematics.

Differences in achievement may be due to student equity issues such as parental involvement and support, learning style, math mindset, access to quality, early education and technology, and learning differences in students such as those with Individualized Education Programs or Other Health Impairments (Tomlinson & McTighe, 2006). Overwhelmingly, the body of research suggested differentiation of discourse as a method to support math learning to meet the needs of all learners (Bloom, 2009; Ensign, 2012; Kapusnick & Hauslein, 2001; Tomlinson & McTighe, 2006; Smets, 2017; Swanson, Ficarra, & Chapin, 2019).

Differentiation is a broad category that encompasses many tools to facilitate learning while meeting the needs of each learner. Differentiation requires a close relationship between educator and learner, fostering a willingness to make observations and modifications that best meet the needs of all on a case-by-case basis. While differentiation is a standard tool in the classroom, primarily through hands-on materials and models, students still show significant variability in retention requiring supplementary practitioner intervention to encourage student success.

The literature further identified talking about math as a high-leverage strategy for growing the capacity for reasoning (Bender, 2013; Costello, 2021a, 2021b; Eddy & Kuehnert, 2018; Ensign, 2012; Wadlington & Wadlington, 2008), explicitly listing communicating about mathematics as a required competency (National Council of Teachers of Mathematics, 2000). The Wisconsin Department of Public Instruction governs the action research setting. It mandates eight Standards of Mathematical Practice K-12, which are "central to teaching and learning mathematics" and include "construct viable arguments and appreciate and critique the reasoning of others" (Wisconsin Department of Public Instruction, 2021, p. 18). Although precise, content-specific teaching language is part of the learning environment, nurturing student collaboration through discourse in mathematics is absent. Therefore, this is an identified discrepancy between recommended practice and instruction methods employed in the classroom.

The question remained, how do discourse strategies impact learner outcomes in mathematics? Can discourse aid learning and encourage students to be actively engaged as math learners? Thus, this research aimed to explore the impact of teaching and encouraging student implementation of differentiated discourse using mathematical prompts on empowerment and achievement in upper elementary children.

Theoretical Framework

When considering how differentiation of discourse strategies in the Montessori elementary classroom might affect math achievement, constructivism is a valuable theoretical lens to process observations and hypotheses. Constructivism asserts that students gain insight when they find a way to create relationships between new ideas based on understood categories or tie novel information to previous knowledge (Dingman, Kent, McComas & Orona, 2019). Constructivism is unique as a learning theory in that it exists as a philosophical explanation of how learners create their own learning rather than requiring the discovery and validation of a learning theory (Schunk, 2012). Constructivists see truth as a flexible concept since each person understands the world through their own experiences. Consequently, constructivism divides into three perspectives according to knowledge construction and types of interactions. Constructivist subdivisions place exogenous, focusing on interactions with the external world, at one extreme with endogenous, focusing on internal construction based on previous constructs, at the other end of the spectrum (Schunk, 2012). A third, middle-ground perspective is dialectical, where knowledge is constructed based on interactions between people and their environments (Schunk, 2012). This third dialectical constructivist perspective, also known as social constructivism, is most helpful in education as it allows for analysis of student learning through manipulating materials and engaging in social interaction (Schunk, 2012). Social interaction contributes to the development of knowledge (Eddy & Kuehnert, 2018); consequently, the action research will focus on social constructivism.

Constructivist theory, supported by work attributed to Piaget and Vygotsky, has numerous recognizable components. For example, Vygotsky's theory of social constructivism surmises that learning occurs in the space between what a person can achieve independently and what they can accomplish with support (Bloom, 2009). Vygotsky further developed the idea of zones of proximal development as the area that exists between what is known and what is not known (Schunk, 2012). The zones of proximal development strengthened the constructivist position that the teacher must engage in individual prompting and guidance tailored to the learner's needs for learning to occur. The teacher gives just enough support to accomplish the new task collaboratively until the student is ready to undertake the task independently, signaling that guidance should be withdrawn (Bender, 2013). Constructivist theory suggests that students build an understanding of new ideas on previous knowledge; thus, the teacher must scaffold lessons to further understanding (Bender, 2013). The teacher must gather information by asking questions with recall-level answers to orient students to a procedure while gathering information and checking for understanding (Eddy & Kuehnert, 2018). This information identifies the lower and upper bounds of proximal development so that the teacher can then incorporate scaffolding strategies to meet the needs of all learners (Schunk, 2012). This use of questioning to find the proximal limits is where the idea of using discourse in differentiation is grounded.

An alternative educational pedagogy, the Montessori Method lends itself well to constructivism. Dr. Montessori emphasized that all children are born with the innate power to create meaning and understanding through activity (Frierson, 2014). Schunk (2012) contends that learning is not passive; students are not empty vessels waiting to be filled. Dr. Montessori aligned herself with the idea of the student as an active participant, responsible for engaging in activity, making interpretations and reflections, and organizing new learning into relevance by relating it to what is already known. Montessori students engage in constructivist theory by partaking in goal setting, monitoring, evaluating their progress, and following their learning schedule. The commonalities between the Montessori Method and constructivism support further socialization and collaborative discourse to support student learning.

If logical thinking is our goal, we must use our whole-to-parts approach. Students must be allowed to question, discover, reflect, and integrate. In short, they must be allowed to construct meaning through collaborative discussion in the Montessori classroom. The teacher is responsible for supporting each learner in this process by engaging in observation to gather information. This information aids scaffolding according to student needs, probing thinking through questioning to open pathways to shared understanding, implementing models and materials to make mathematics visual and allow for applying and analyzing, and encouraging reflection and justification of strategies (Eddy & Kuehnert, 2018). This research will explore the efficacy of differentiating discourse methods to support math learning through the lens of constructivism.

Review of Literature

Once students enter the upper elementary Montessori classroom, math performance becomes an area of statistical interest. Although each class may seem homogeneous, the measurement of mastery of math skills shows a heterogeneous group. Learner needs vary based on various factors such as ability, culture, race, economic background, gender, life experience, motivation, interest, learning styles, and support systems (Tomlinson & McTighe, 2006). While students can close achievement gaps, learn, and seek challenges through the benefits of differentiated instruction across all learning subjects, the variability of student success and levels of abstraction of math concepts necessitates special attention to move all learners forward. Effective teachers differentiate instruction to meet the needs of each individual in a way that allows them to maximize their potential (Bloom, 2009). Ensign (2012) built on this idea in a recent article expressing the learning goal as looking beyond mere growth, hoping that every student moves to a much higher level of understanding. The purpose of this literature review is to explore if and how offering differentiated discourse strategies through student collaboration impacted learner outcomes. This literature review will include a trend toward discourse, why education needs differentiated discourse, and what differentiation looks like in the classroom.

A Trend Toward Discourse

Eddy and Kuehnert (2018) followed the historical importance of questioning in education. Their article showed that inquiry first drew attention as a teaching model in the 1960s and 1970s but mainly asked for recall and procedural answers. The 1980s and 1990s saw a shift to Socratic seminars and shared understanding through student creation of knowledge (Eddy & Kuehnert, 2018). Research of the literature revealed discourse to be an apt differentiation target for math (Anderson-Pence, 2015; Bender, 2013; Costello, 2021a; Costello 2021b; Eddy & Kuehnert, 2018; Ensign, 2012; Jansen, 2012; Kapusnick & Hauslein, 2001; Moncada & Riggs-Woessner, 2019; Sharma, 2015; Wadlington & Wadlington, 2008). The National Council of Teachers of Mathematics (2000) called for communication as one of ten Standards for School

Mathematics: Prekindergarten through Grade 12 (pp. 60-63). The communication standards for mathematics are as follows: to organize and consolidate mathematical thinking, communicate mathematical understanding clearly to others, analyze and evaluate the mathematical knowledge of others, use mathematical language to express mathematical ideas accurately, and develop questioning and inquiry (National Council of Teachers of Mathematics, 2000).

To meet the goals set forth by the National Council of Teachers of Mathematics (NCTM), students must have opportunities to engage in mathematical conversations and use inadequate solutions as learning opportunities, shifting the focus from grades to constructive feedback. Scharma (2015) explained this as a shift in our approach to teaching and learning math to meet the needs of our society, in which teachers must facilitate discourse and conceptual development. Students who can communicate about math consolidate their thinking through accountable talk, sharing ideas, and listening to others, which leads to alternate interpretation, making connections, giving multiple pathways to shared understanding, and refining their stance (Costello, 2021a; Eddy & Kuehnert, 2018).

The teacher facilitates mathematical discourse as a high leverage strategy to grow the capacity for reasoning. Students who engage in questioning mathematics become better problem solvers and critical thinkers (NCTM, 2000). Teachers must identify a goal, look at student work, analyze the work and its relationship to the teaching, and reflect on the implications (Anderson-Pence, 2015).

Like any other differentiation strategy, teachers begin by probing to uncover prior knowledge before encouraging student inquiry through finding creative ways to approach problems and linking student interest to the task at hand (Costello, 2021a). In Montessori upper elementary classrooms, mathematics becomes more abstract and complex, requiring increased problem-solving (Bender, 2013). The gap grows wider between students who understand concepts quickly and those who need support (Bloom, 2009). It is vital to have a foundation for the construction of knowledge through discourse.

The Need for Differentiation

Differentiation is required to support meeting the needs of all learners (Tomlinson & McTighe, 2006). The predominant classroom model in the United States includes learners of mixed-ability grouped due to various factors, including their academic potential, learning readiness, and social development (Bloom, 2009). Additionally, differentiation in homogeneous and heterogeneous groupings increases student engagement, fosters acceptance of diversity, and improves instruction (Kapusnick & Hauslein, 2001). Consider that the inclusion model of learning places most students with special needs in the general classroom (Bloom, 2009). The Common Core State Standards and legislation such as the No Child Left Behind Act or Every Student Achieves Act set standards for proficiency for all students. These regulations include those with special needs, learning disabilities, limited English language skills, living in poverty, and high-achieving students (Bloom, 2009; Swanson, Ficarra, & Chapin, 2019). Swanson, Ficarra, and Chapin (2019) noted that gifted and talented students require "differentiated instruction that focuses on rigorous advanced content, engages creative and critical thinking skills, problem-solving, sophisticated processing, and interdisciplinary learning" (p. 118). Differentiation is a teaching asset that should be part of every classroom to meet the needs of all learners.

What Differentiation Looks Like in the Classroom

Sir Francis Bacon outlined the scientific method: observe a problem, create a hypothesis, test the hypothesis, then draw conclusions to inform further testing or share with peers. In much the same way, teachers accomplish differentiation as a support for learning by observing, reflecting, monitoring, and adjusting instruction and material use (Bloom, 2009). Bender (2013) highlighted the essential instructional skill of knowing the needs of the individuals before providing support. Within the Montessori method, this approach to differentiation is a central pedagogical tenant known as following the child. Bloom (2009) notes that effective teachers take on the responsibility to adapt rather than ask the students to assimilate to instruction. A teacher who is responsive to the needs of the individual in terms of readiness, interests, and learning styles, allows their students to maximize their strengths (Kapusnick & Hauslein, 2001). In keeping with Vygotsky's zones of proximal development theory, the goal is to take each child beyond their comfort zone, just short of the point of frustration (Kapusnick & Hauslein, 2001).

Teachers have access to multiple processes for differentiated instruction. Smets (2017) described high-quality, differentiated instruction with the following four components: knowing their students, being aware of teaching goals, building on what the child already knows and can do, then differentiating instruction through variable modeling, using flexible grouping, and providing maximum choice. Bloom (2009) also encouraged analyzing learning styles and formative assessments before building learning centers.

Kapusnick and Hauslein (2001) looked specifically at gifted and talented learners' needs. They mentioned independent learning centers and these seven other intervention strategies: acceleration, curriculum compacting, separate study flexible grouping, tiered activities, contracts, and complex questions. Wadlington and Wadlington (2008) came from the other end of the learning spectrum in thinking about disability. They suggested mathematical communication in multiple ways, emphasizing vocabulary tied to concrete examples, explaining thinking, and verbalizing signs and symbols. They stressed leaning over rote memorization as one of the fundamental intervention strategies to help students with mathematical disabilities to succeed.

Costello (2021b) narrowed the lens set by Kapusnick and Hauslein (2001) in that he specifically encouraged discourse strategies through repeating, paraphrasing, adding on, and providing wait time to construct ideas. He also advocated setting clear expectations for conversations, using the think/pair/share strategies, and asking for written work to document their thinking in various forms such as journals, entrance/exit slips, free writes, and note-taking. With Kapusnick and Hauslein recommending questioning for gifted and talented learners, Costello encouraging discourse strategies, and Wadlington and Wadlington emphasizing multiple types of communication, a growing trend emerges calling for discourse as a math differentiation tool.

Classroom discourse around mathematical ideas differentiated for learners requires social interaction. The Constructivist perspective holds that students can develop deep knowledge through instruction in prerequisite skills and support from their teacher as a facilitator. At the same time, they work to understand meaning further and construct knowledge through using materials suitable to their level of skill and expression of their understanding to lead to deep, conceptual understanding (Bender, 2013). Eddy and Kuehnert (2018) emphasize that social interaction contributes to knowledge development. As a constructivist system that focuses on the child, Montessori gives opportunities for individual prompting and provides guidance tailored to the learner's needs. The level of support and guidance is in line with Bender's (2013) prescription to give just enough support to accomplish the new task until the student is ready to undertake the work independently and then remove guidance. The result is that the apprentice becomes the expert.

The educator examines the best ways to support math learning through differentiation and implements differentiation through scaffolding. Bloom (2009) described differentiation as building on what each student already knows and ushering them to the next level of learning through exploring information and expressing learning. Bloom's description is akin to the critical characteristics of scaffolding, as summarized by Janneke van de Pol, Monique Volman, and Jos Beishuizen (2010). These vital characteristics include contingency, where the teacher has evidence that a student needs mentor support due to the learner's present understanding, which fades when the teacher gradually withdraws support as the learner constructs understanding. The transfer of responsibility only happens when the child can progress independently. Scaffolding fits well with Montessori as a constructivist theory, in which teachers gradually withdraw support as students construct their knowledge.

Differentiation in the classroom allows natural mentorship between students. Mentorship is a function of the multi-age classroom and is also a natural extension of the social-emotional nurturing of character traits like empathy and community. The pairing of a more capable peer with a student of lower cognitive understanding is one way to scaffold conceptual knowledge (Eddy & Kuehnert, 2018). Ensign (2012) suggested self-evaluation statements that culminate in readiness to instruct. A student who feels that they can teach solidifies their learning by engaging in mentorship. Teachers can easily assign partnerships in the Montessori classroom due to the broad range of ages and abilities.

A Montessori classroom's broad range of ages and abilities results from the traditional three-year mixed-age groupings. Dr. Montessori divided students into planes of development based on observations of characteristics that children exhibited, which resulted in her designing environments and materials to meet the formative needs of the children in each stage. Dr.

Montessori spoke of four Planes of Development: 0 to 6; 6 to 12; 12 to 18; and 18 to 24 (The American Montessori Society, 2021). New sensitive periods and changing psychological characteristics dictate a meaningful and successful educational program for each plane. The multi-age classroom spanning three years is one of the five standard components according to The American Montessori Society (2021) for high fidelity implementation for any Montessori classroom.

The literature supports the differentiation of classroom discourse by combining different groups for different purposes. Ensign (2012) described how success in mathematics was due to flexible grouping and differentiating lessons through a combination of teacher-directed small-group work and heterogeneous partnerships, which involved peer teaching. In this way, students that need the most support get it, and the others can explore or seek enrichment.

Similarly, Gentry and Owen (1999) found that cluster grouping of students identified as gifted, high achieving, or high-ability positively impacted the cluster group and positively impacted others. Gentry and Owen (1999) discovered that teachers were better able to meet the needs of the other students and these students were able to find their voice through confidently guiding others. Cluster grouping had the effect of reducing the range of achievement levels among students. In addition to recognizing the advantages of mixed-age grouping and flexibility for teaching within the scope of learners of that age, Dr. Montessori also recognized the importance of providing concrete representations for abstract ideas (Montessori, 1912).

The literature on differentiation for mathematics concludes that concrete manipulatives are necessary when introducing concepts and should also be available later in elementary grades to lend a tangible representation to abstract concepts. Bender (2013) and Costello (2021a) advocate using manipulatives to represent thinking through problem-solving, helping students achieve a level of independent mastery that suits their needs in their current stage of development.

Conceptual mathematics is a model commonly used in traditional education today that dictates visual models combined with language and discourse to examine relationships between mathematical concepts, rather than procedures and memorization, to find solutions. Montessori aligns with this ideal and includes concrete manipulatives, many implemented over nine years of classroom instruction, serving different purposes based on the child's needs. The masterfully simplistic and attractive Montessori materials inspire repetition and the internalization of abstractions through concrete interaction. The Montessori educator knows that the materials are to be manipulated by the hand during a child's early ages to create mental muscle memories that extend into the upper grades when math becomes more abstract. Moncada & Woessner (2019) looked at Dr. Montessori's views on math absorption and highlighted that the use of materials is much more than manipulatives and requires the teacher to supplement their use by articulating the concrete before moving to abstraction. A wise teacher needs to observe the child and recognize when the time is right to use the materials to make that transition.

Observation is a critical piece of differentiation. The literature supports that the teacher must gather information before differentiating questioning. Eddy and Kuehnert (2018) mentioned that teachers use questioning to provide procedures to follow while they collect the information that assists them in managing the needs of the group and helping students organize their work. Dr. Montessori made methodological observation an essential component of her method (Montessori, 1912).

Further, there must also be time for reflecting on observations for the educator and the learner. Bloom (2009) found that successful differentiation occurred with greater degrees of purposeful reflection, followed by the implementation of individualized instruction. Students consider whether they could teach the concept, do it independently, do it with help, or not understand it (Ensign, 2012). Student reflection requires them to justify their strategies in other contexts (Eddy & Kuehnert, 2018). As a scientific method, Montessori pedagogy also employs observation, reflection, and a prepared environment that changes to meet the needs of learners. Teacher reflection leads to further differentiation based on student work making this a collective task.

One last contributing factor to quality differentiation practices is the natural development of strong relationships. Student differentiation is most successful for teachers who know their students well and understand their prior experiences and home support (Bloom, 2009). Montessori pedagogy is no different. Strong relationships are a product of teaching the same child and supporting the same family over three years. Solid relationships and contextual knowledge between students and families help the observations made and impact reflections while also informing instruction.

Discourse Methods

Discourse methods vary broadly, mirroring the diversity of the learners they serve. Bender (2013) advocated for representative examples rather than rote memory, examining patterns, and using mnemonics. Learning essential vocabulary is crucial for solving problems, so teaching cue words and differentiating internalizing these code word indicators that help represent mathematical ideas leads to student success. To aid visualization, Bender gave specific collaboration strategies like games and buddy activities, think-aloud activities, cueing in steps, and concrete manipulatives. Costello (2021a) echoed Bender's ideas by suggesting discourse opportunities that include whole-class, small-group, pairs of students, and student/teacher discussions. A combination of options can happen concurrently. A productive conversation can consist of a teacher facilitating guided practice problems. Teachers lead the conversation for students who need more support. Other heterogeneous pairs discuss the guided practice problems together before meeting with the teacher, who then goes over the same steps, but at a much faster pace (Ensign, 2012). Costello's student/teacher discussions pair well with Bender's push for using cognitive guided inquiry methods to aid students in visualizing processing problems, which bridges the gap between understanding and abstract problem solving by focusing on pertinent information. To effectively participate in this sort of questioning, educators must know the context of this discourse.

Conclusion

Discourse that leads to visualization allows students to connect conceptual ideas, structural procedures, and real-world applications (Eddy & Kuehnert, 2018). Discourse is a reliable tool to support this goal in mathematics. Examining students' thinking allows teachers to gain knowledge and awareness of student needs and, through reflection, more informed choices in teaching (Anderson-Pence, 2015). Furthermore, systematic examination of student thinking in mathematics increases awareness of student needs in other areas (Anderson-Pence, 2015). There is no single journal article about the effects of differentiation of discourse for mathematics in the elementary Montessori classroom; thus, research in this area would be prudent to the broader Montessori and educational community.

Differentiation of discourse in any classroom requires that educators have the time necessary to plan and collaborate with peers to do it well. Bloom (2009) revealed that the teachers who participated in differentiated instruction intentionally made time for purposeful collaboration with colleagues. While Bloom and Anderson-Pence (2015) agree that ideal instruction includes differentiation, having time for teacher collaboration is not the only critical factor. Still, teachers must access professional development resources to support creating change by examining student thinking.

Educators need a fundamental understanding of examples and levels of discourse strategies as a vital first step. Teacher instruction in public Montessori programs must align with Common Core Standards. Educators are held accountable for student learning through standardized testing, yet standardized testing does not accommodate student differences (Bloom, 2009). Swanson (2019) explained that students in the same classroom are not learning the same way, and core standards do not dictate what to teach or how. Bloom (2009) found that teachers were reluctant to engage when they did not feel fully confident, and Anderson-Pence (2015) also expounded on the uncertainty their participants experienced in getting started. Educators require specific training in this area to feel optimistic about implementation.

Training in using discourse strategies in the classroom goes beyond information sharing. As Saunders (2013) pointed out, continuing professional development is required to grow Instructional Intelligence. Instructional Intelligence is the combination of the art and science of teaching, where science is the knowledge and art is the skill of the teacher's practice of applying this understanding to measuring and assessing data collected to differentiate instruction to meet the needs of the individuals. The teacher employs this learning to merge "curriculum, assessment, knowledge of how students learn, instructional skills, tactics and strategies and theories of change (Saunders, 2013, p. 311)." Professional development that supports systematically examining student thinking and provides tools to process information and enhance communication skills to have productive interactions must be part of teacher education.

In addition to having time for collaboration and planning instruction and background information on how to implement discourse differentiation strategies, teachers also need guidance in their work. Ensign (2012) described how the school models in his research on successful differentiation received support from a new principal, math coach, and collegial teaching staff to encourage teachers to develop effective differentiated teaching strategies as they learned from one another. Both schools demonstrated success, but ultimately, budget cuts eliminated coaches and slowed differentiation. Most schools operate with tight budget constraints, making such support staff sparse.

By analyzing the information about differentiation, it is observable that discourse is the most potent supportive intervention strategy to support learner outcomes. This intervention is lacking in traditional and alternative educational approaches, including the Montessori Method. Research which aims to fill the void between the need for differentiation of discourse and its efficacy in a Montessori learning environment would be helpful when looking at the broader educational community and best practices for positive learner outcomes.

Methodology

This study used an experimental design to examine the impact of discourse in improving learner outcomes on problem-solving in mathematics. Multiple data collection methods were employed to obtain triangulation and included artifacts, observational, and inquiry data. Inquiry data came from a survey as a pre-and post-rubric (Appendix A) to indicate quantitative scores for the overall perception of classroom discourse levels. Pre-assessment and post-assessment formative assessment artifacts were collected to give quantitative evidence for change, if any, in the skill level demonstrated in solving word problems.

Additionally, student work submissions provided quantitative data on the accuracy of answers. Student work submissions showed qualitative data as visual representations of thinking. Student-generated observational data as tally marks also accompanied student work submissions to show discourse and reflections throughout the problem-solving process (Appendix B). Furthermore, the lead teacher collected quantitative data by maintaining a tally log of discourse strategies used when going over problems with the students (Appendix C). Lastly, a research journal of implementation also provided qualitative data about the implementation process.

The population for this action research study was an upper elementary, suburban, public Montessori classroom in the Midwest Region of the United States. The total student population of the Montessori school was just under 200, with a minority enrollment of 16%. The sample classroom served 28 upper elementary students. Four of the oldest students attended advanced math classes outside the research setting, bringing the total number of study participants to 24, comprising 11 female students and 13 male students. Quantitative data collected from summative math assessments and standardized tests demonstrated that 40% of students scored under the proficient level in math during the previous school year.

The intervention process began with a rubric that examined perceived empowerment in math learning, completed by both the teacher and the students. Due to the constructivist nature of the learning environment, it made sense to examine student empowerment in learning. Hufferd-Ackles, Fuson & Sherin (2015) provided a discourse rubric for data collection (Appendix A) before and after the research intervention. The rubric asked students to grade each component on a one to four rating scale. The lowest marker indicated traditional, teacher-directed classroom characteristics and moved developmentally toward level four, where students facilitate their math learning through discourse with teacher support. The component areas for ranking included the teacher's role, questioning, explaining mathematical thinking, mathematical representations, and building student responsibility within the community. Notions of self-efficacy in mathematical discourse were valuable to inform further instruction, expectations, and adaptations.

Students also completed a pre-and post-intervention assessment to supplement the self-efficacy rubric. This assessment provided evidence of learning and refined implementation of the intervention and informed instruction. Students completed the pre-and post-assessments as a packet of word problems during a 45-minute quiet work period. The teacher scored and recorded these grades but did not share them with students.

Students independently completed packets of word problems given weekly over four weeks following the pre-assessments. The fourth-grade students completed six single-step, static and dynamic addition and subtraction word problems as far as the 100's place each week. The fifth-grade students completed six single-step, static (no exchanging or carrying) and dynamic addition (with exchanging or carrying), subtraction, multiplication, and division word problems as far as the 10,000's place. The sixth-grade students completed four multi-step word problems, which required various combinations of operations, and employed fractions and decimals, no farther than the 100's or 0.01 place. The teacher did not score these packets before student collaboration within the intervention process.

Next, students were instructed on collaboration expectations and participated in their first discourse collaboration session. In tandem with implementing student-completed word problems, students spent 30 minutes collaborating with their peers while using the red cards from Curriculum Associates, LLC. (2022). The red cards contained 'sentence starters' designed to support math conversations supporting collaborative work. Students worked with grade-level partners, compared answers, and discussed possible approaches and solutions using the red cards as models for discourse.

In addition, students also collected observational data (Appendix B), noting via tally-mark each time they used or heard a discourse strategy used during peer collaboration. Furthermore, students completed open-ended reflections following their collaborative experiences. The observational data collected as tallies helped answer whether the intervention had a positive, negative, or neutral effect on occurrences of discourse usage and provided insight into the students' experiences. It also allowed students to become active participants in the research, which developed a shared responsibility for the research undertaking.

Following student collaboration, the teacher scored the packets. The teacher gave scores as a percentage of correct answers and a separate score for showing work. A score of one indicated no or little work shown, a score of two indicated some work was shown but was incomplete, and a score of three indicated a full explanation of thinking and labels with answers.

Then students met with the teacher by grade level and went over the problems together. The teacher kept a tally log of the inquiry questions used from the blue cards from Curriculum Associates, LLC. (2022). These questions asked students to reason, explain, and critique their answers. After one week, I decided to omit this teacher-centered piece of the action research due to its lack of compatibility with the focus on student discourse. The intervention concluded with a second completion of the Hufferd-Ackles, Fuson, & Sherin (2015) discourse rubric. This data tool identified what change there was in the notion of self-efficacy in classroom discourse after this intervention.

The intervention lasted a total of six weeks. One week was to administer the self-efficacy rubric and baseline story problem assessment. Four weeks spanned the intervention when students independently completed word problem packets and collaborated with peers using the red cards from Curriculum Associates, LLC. (2022) and verified the correct solutions facilitated by teacher support with the blue cards from Curriculum Associates, LLC. (2022). One week was dedicated to post-assessment on story problem solutions without discourse to identify whether the discourse process led to individual learning. The study concluded with the self-efficacy rubric form. At the end of the six weeks, the data illuminated trends and variabilities correlated with learner characteristics.

Analysis of the Data

This study examined the impact of teaching, modeling, and using discourse strategies on student perceptions of math learning and learner outcomes. Qualitative and quantitative data tools facilitated the collection of various types of data. An overall picture emerged demonstrating a few key findings.

This study's first research question asked about the collective perception of student self-efficacy for collaboration in classroom math discourse. Twenty-five respondents (24 students and the teacher) completed a survey that gave quantitative data to five questions. The rubric from Hufferd-Ackles, Fuson, & Sherin (2015) provided inquiry questions (Appendix A) to establish a baseline for measurement before beginning the intervention and a concluding survey for comparison. Each of the five questions inquired about perceptions regarding responsibility for aspects of learning surrounding the teacher's role, questioning in the classroom, explaining mathematical thinking, making mathematical representations, and building student responsibility within the community.

The five questions each included an answer as a ranking on a four-tiered scale starting from identifying a teacher-centered classroom to a student-centered classroom environment. The discourse study data included the collection of 125 responses from the 25 respondents to five questions on two separate occasions; before the action research study and after the intervention. Student responses were entered into Excel, totaled according to place on the teacher-centered to student-centered classroom continuum, and analyzed to determine the starting and ending point of the action research study.

Figure 1

Perceived Levels of Discourse Responsibility Before and After the Intervention

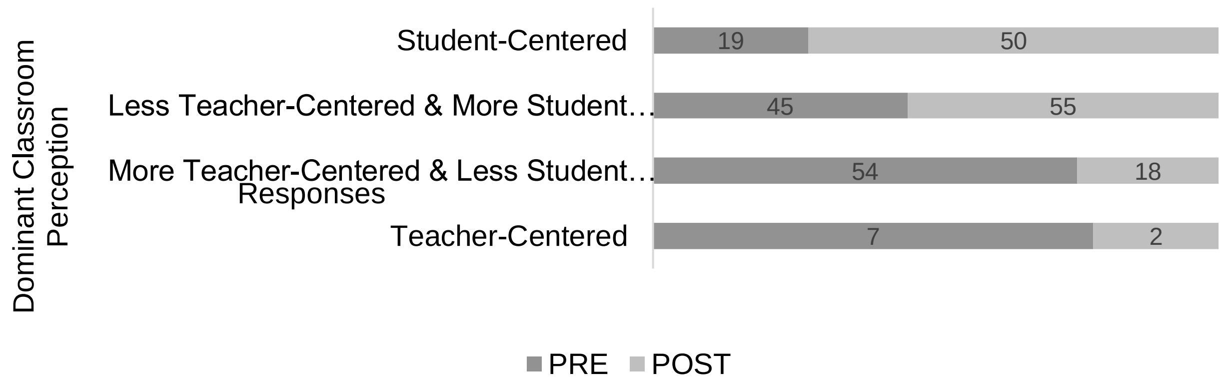


Figure 1 indicates that the children and their teacher viewed their classroom discourse as more teacher-centered than student-centered before the intervention. The trend shown in dark gray bars indicates that 54 respondents considered the classroom more teacher-centered and less student-directed. In contrast, 45 thought it to be less teacher-centered and more student-centered, and 19 indicated a student-centered classroom, while only seven specified a teacher-centered learning environment. The data affirmed a need to assess students' perceptions of self-efficacy. This data demonstrated the requirement to empower students in mathematics to align student-centered discourse with pedological goals.

The pre-assessment findings contrasted with the post-assessment survey results. Participants answered the same inquiry questions examining feelings of autonomy in math collaboration.

They ranked them along the student-centered to teacher-centered continuum, but this time in response to the intervention experience. The light gray bars in Figure 1 show a stark contrast in data, indicating that the overall perception of responsibility for discourse over the intervention was more of the students' responsibility. The majority of responses, 105 out of 125, supported student-centered or more student-centered feelings, while the minority, 20 out of 125, perceived the classroom to be teacher-centered or more teacher-centered. The total number that viewed the classroom as primarily teacher-centered dropped from seven in the pre-assessment to two in the post-assessment. The overall result of the intervention was an increased feeling of empowered action in mathematical conversation around problem-solving.

In addition to identifying the perception of student ownership and self-efficacy in math discourse, the intervention yielded quantitative data that has to do with externally measurable math learning and objectives. Students independently completed word-problem packets as pre- and post-assessments. Over four weeks, students engaged in collaborative discourse with peers, using the introduced strategies to complete word problem packets. The scores compiled over the six weeks of the action research study show student progress.

Figure 2 shows the scores for the pre-and post-assessments and scores from the word problem packets over the four-week intervention. Table 1 breaks that data into the mean, median, upper, and lower quartile numbers.

The mean data show a four-point improvement, and the median is identical from the independent completion of the pre-assessment to the post-assessment. The chart demonstrates a more robust set of scores over the four weeks of the discourse-implemented collaborative assessments. The interquartile range gave the spread of the middle 50% of scores so that outliers would not skew the data. While the post-assessment score showed a nominal increase in the mean score and a static median score, the interquartile bottom went up by four percentage points, and the interquartile top went up 12 percentage points, showing an improvement from the pre-assessment to the post-assessment.

Figure 2

Assessment Scores Across the Intervention

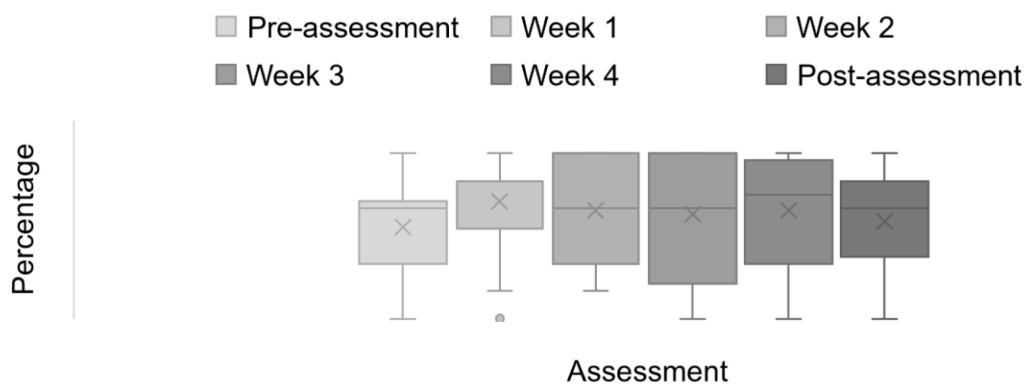


Table 1

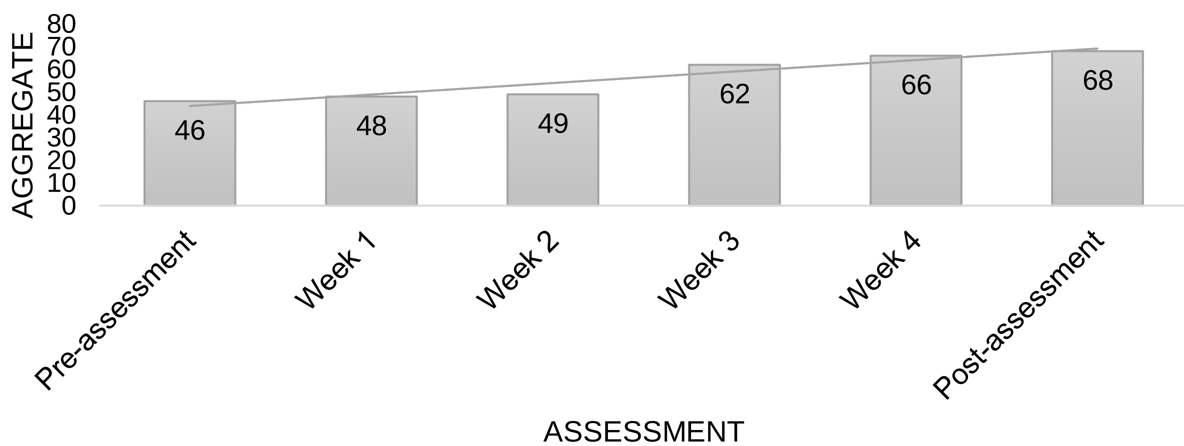
Box Plot Breakdown

Assessment	Mean	Median	Interquartile Top	Interquartile Bottom
Pre-assessment	55	67	71	33
Week 1	71	71	83	57
Week 2	65	67	100	33
Week 3	63	67	100	21
Week 4	66	75	96	33
Post-assessment	59	67	83	37

In addition to success in precision with mathematics in finding correct answers, the data revealed an unexpected variable. There was a significant increase in student work submissions for modeling mathematical thinking throughout the intervention. The score for student work submissions showed work starting at a level one for showing none or very little of their thinking, to a level two for displaying some of their thinking, and culminating in a three for showing their thinking and including word labels with their answers. The trend line in Figure 3 shows a steady increase throughout the study.

Figure 3

Mathematical Modeling



The aggregate score on the pre-assessment was 46 for 24 students, which would average out to all students scoring 1.9, meaning collectively that the class represented very little of their work. Additionally, only three of the 24 students added word labels to their answers on the pre-assessment. The aggregate score for the post-assessment was 68, resulting in an average of 2.8, meaning nearly all students showed their thinking. After reviewing the post-assessments, 19 of 24 students included word labels. This finding was relevant as the location of the action research student is within a state which lists "model with mathematics" as one of the standards for mathematical practice (Wisconsin Department of Public Instruction, 2021, p. 18) in public schools.

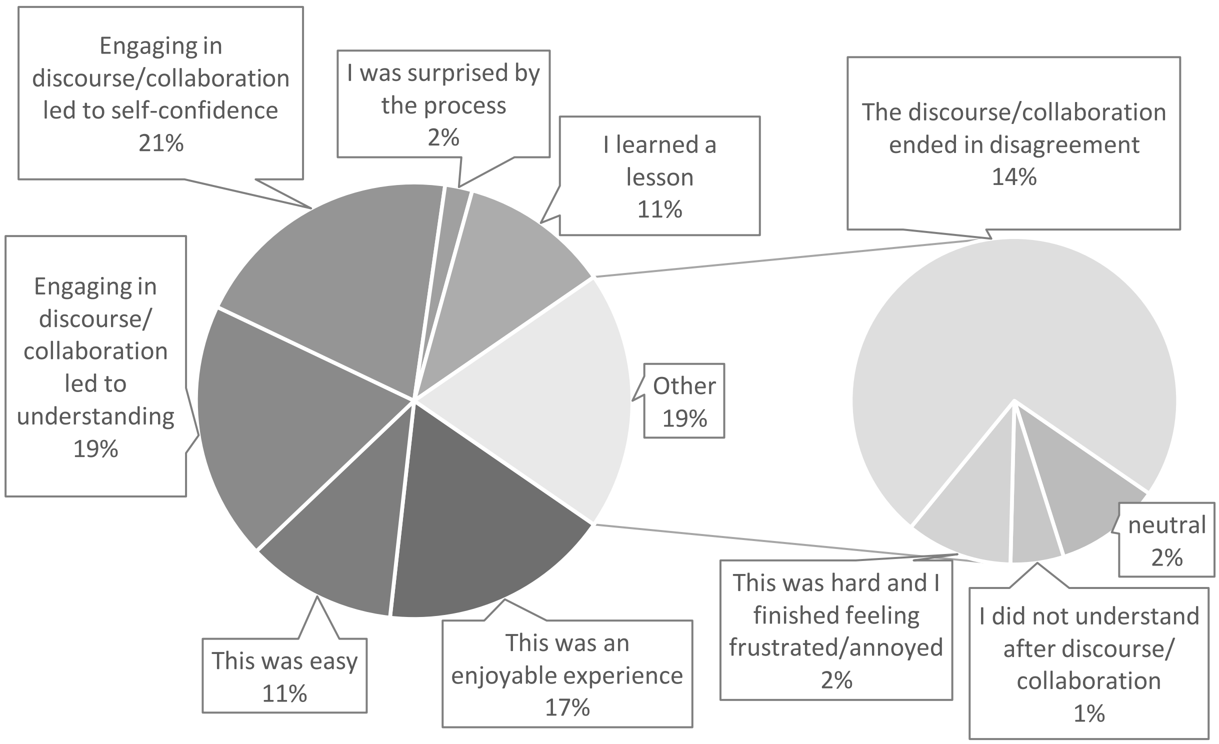
The action research study revealed quantitative data about learner outcomes and modeling with mathematics and identified the students' preferences for discourse strategies (Appendix B). According to the red cards from Curriculum Associates, LLC, the data showed a few frequently used techniques. (2022). Students recorded, via tally marks, 1,864 instances when a student used a sentence starter or heard one used by a collaborating peer.

Sentence starters support collaborative math conversations; the student choices show preferences. The sentence starter, "I disagree with…," showed 552 student tally marks, followed by "I agree with…" having 271 tally marks, and then "I started solving the problem by…" with 268 student tally marks. These three sentence starters carried the majority of student choices, with the 4th most used, "I noticed a connection between…," only carrying 95 tally marks. This trio of sentence starters opens the conversation and then facilitates the transition from one speaker to the next. The permission to politely disagree or agree and build off a peer's statement led students to describe their thinking and use evidence to support their ideas. Students could also use this form to give qualitative data through an invitation for open-ended reflection.

Qualitative data was collected from the open-ended student reflections (Appendix B) and coded according to themes. Twenty-four students participated in the four-week intervention resulting in 96 reflection forms. Two separate students were absent for one of the weeks of the intervention, missing both the independent problem solving and the discourse/collaboration activity. Two of the forms outlined student thinking on the problem solving but did not ascribe a positive, negative, or neutral attribute to the process. Considering absences, non-specific responses, and 18 blank reflections, 81% of all forms contained codable student reflections. Student reflections showed 81% positive perceptions, 17% negative perceptions, and 2% neutral. Figure 4 breaks these percentages down by the specificity of coded student reflections.

Figure 4

Student Reflections Breakdown



The "I learned a lesson" response indicated a broad range of ideas from students that included the following: always check your work, use multiple strategies, listen to how others solve problems to grow to learn, listening is hard when not in agreement with others, explaining thinking is complicated and needs work, it is essential to focus on details and read the problems thoroughly, use materials when necessary, put in the best effort, and use active listening to focus. As evidence of deep reflection on student participation in the learning process, these statements indicate social learning.

Data assessment revealed three main relevant themes. The first theme came from the quantitative data collected from the pre-assessments, word problem packets, and post-assessments. The data indeed demonstrated a statistically insignificant improvement of correct answer percentage scores from the pre-assessment, through the four weeks, to the post-assessment in terms of median and mode. However, the interquartile top and bottom ranges showed increases over the six weeks of the research, indicating student success through collaboration. The nominal numerical data notwithstanding, reflections on student behavior and participation pointed to student learning as relevant to overall mathematical success. It is also noteworthy that the data collected from the packets showed an unexpected improvement in students engaging in mathematical modeling over the six weeks of the action research study. Students nearly improved a whole data point (0.9) in showing their work on a 3-point scale, and 12.5% of students showed word problem labels as part of their answers on the pre-assessment, which increased to 79% doing so on the post-assessment.

Learner outcomes aside, the central theme is a considerable shift in perceptions surrounding responsibility for discourse from teacher-centered to student-centered. Before the intervention, 49% of the responses classified the classroom discourse as teacher-centered and more teacher-centered. Afterward, 84% of responses ranked the classroom as student-centered and more student-centered, showing a tremendous increase in student responsibility for participation in the learning process. Closely related to this shift in perceptions, the final key finding suggested that the overall experience positively impacted students. Most of the class (81%) responded affirmatively to the discourse collaboration process. These three indicators show that instruction and engaging in discourse collaboration is a significant component of mathematical learning in the classroom for upper elementary students.

Action Plan

This six-week action research study examined the outcome of the implementation of presenting students with discourse strategies, modeling their use, and providing opportunities and expectations for math discourse. Solving word problems offered the chance to focus on engaging in mathematical conversations independent of instruction of new skills. The study measured learner outcomes regarding perceptions surrounding responsibility for math learning and accuracy of responses and provided qualitative and quantitative data.

Results

While the data pointed to a statistically insignificant improvement in the accuracy of answers throughout the study, it is relevant that the interquartile top and bottom ranges both showed improvement. The maximum capacity of the interquartile data would represent the higher-achieving math learners, who showed improvement between the pre-assessment and post-assessment. In contrast, the minimum range would mean those who struggled the most and showed improvement. As the literature suggested, this increase indicates that student engagement in discourse surrounding mathematics does serve to close achievement gaps while also meeting the needs of all learners.

Although the data showed a numerically insignificant improvement in the accuracy of answers, the study is relevant in this learning environment. Even though this was a six-week study, the intervention lasted only four weeks. After just four weeks of practice, there was a modest improvement in scores, indicating that the gain may be more substantial over three years in the upper elementary learning environment, with consistent student discourse in mathematics implementation. This increase in the data scores combined with insightful student responses in the open-ended student reflections section (Appendix B) is in line with the literature. It corroborates the assertion made by NCTM that engaging in questioning mathematics leads students to become more accurate in problem-solving and enhances critical thinking skills.

The body of literature not only identified talking about mathematics as a critical skill (NCTM, 2000) but also expressed the importance of modeling with mathematics as a critical standard for mathematical practice (Wisconsin Department of Public Instruction, 2021). With this in mind, the data revealed an unexpected and fortunate outcome of the intervention. Analysis of the student submissions yielded responses that improved considerably over time regarding the application of mathematical modeling. The average of the aggregate scores demonstrated a shift from a minority of students showing their work, or revealing very little, to the majority of the class showing their thinking in recognizable, successive steps. Student instances of word problem labels added to answers went from nearly nonexistent to the class norm. These data outcomes show the power of student mathematical discourse on mathematical modeling.

Recalling the theoretical framework for this study and the adherence of Montessori educational pedagogy to social constructivism, this study's most relevant quantifiable outcome was the impact on student mindset, behavior, and participation. The students engaged in social interaction, either expressing their understanding or seeking assistance by analyzing and evaluating their peers' mathematics understanding while participating in mathematical discourse, contributing to knowledge development. The spectrum of discourse choices provided differentiation in that each child had access to the language necessary to create meaning.

Paramount to social constructivism is the idea that learners create meaning through their experiences and interactions in the world. Furthermore, social constructivism adds manipulating materials and engaging in social interaction as contributing factors in building knowledge. This type of learning is active and requires student participation. The learner has a tremendous responsibility for learner outcomes.

The most relevant learner outcome revealed in the study was a shift in student perceptions of who is responsible for the mathematical discourse in the classroom. The study outcome uncovered that engaging in mathematical discourse had a favorable effect on student self-efficacy. The study also indicated that students found engaging in mathematical discourse a profitable experience. The broad majority of study participants, 81%, reported in favor of engaging in discourse, and students introspectively shared a variety of lessons learned. Students feeling empowered as learners and having a positive perception of their work is essential in a social constructivism learning model.

Limitations of the Study

This study employed interactions between grade-level peers. In keeping with the goal of differentiation, recommendations in the literature, and Montessori pedagogy, this action research project might have revealed different outcomes if students engaged in non-grade-level-specific interactions. One possibility could include mixed-age groupings to engage in the discourse to see if mentoring relationships enhanced, diminished, or had no effect on outcomes. Another option would be to engage in cluster grouping of students by creating cohorts of similar abilities, as the literature suggested, to examine whether cluster grouping reduced the range of levels.

Social constructivism requires that the educator observe and question students to identify their upper and lower bound of proximal development. This zone between the upper and lower identifies the space between what children can do with support and what they can do independently. This study intended to include teacher questioning and modeling; however, I removed this component after limited employment as it went against the focus on student discourse. A more long-term and encompassing implementation of this study, including the adult identifying the upper and lower bound of proximal development to guide differentiation of instruction, may impact outcomes.

Recommendations

Knowing that math in upper elementary is more complex and abstract, coupled with noticeable gaps in math competency, educators have to use differentiation skills to meet their learners' needs and achieve goals established by the school or district. Practitioners should intentionally instruct students in using a variety of discourse strategies to build student skills and understanding. Students use the appropriate tactics to open conversations, clarify their learning, see things from the perspectives of others, respectfully disagree, and consider multiple modalities. Engagement in this work results in increased accuracy of student responses for the largest group of lower range scorers and the largest group of upper range scorers. Additionally, collaboration through mathematics discourse contributes to a dramatic increase in student modeling of mathematical thinking. Students learn lessons beyond the accuracy of answers by engaging in these collaborative learning conversations. Most importantly, this undertaking enhances student participation and results in most students feeling worthy and self-efficacious in their learning. Considering that math competency is essential to success in higher education and life, it is easy to conclude that explicit student instruction on engaging in mathematics discourse is worthwhile, advantageous, and would benefit the broader educational community.

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Appendix A
Rubric from Hufferd-Ackles, Fuson, & Sherin (2015) Pre- and Post-Survey Questions

Levels of Discourse Rubric

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students believe that their ideas are acceptable by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Hufferd-Ackles, Fuson, Sherin (2004)

Levels of Discourse in Our Classroom

Choose the selection that best fits each question.

How would you best express the teacher's role during math instruction and work? *

- Teacher is at the front of the room and dominates conversation.
- Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.
- Teacher facilitates conversation between students, and encourages students to ask questions of one another
- Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.

What does questioning look like in our classroom during math? *

- Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.
- Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.
- Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.
- Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.

What does explaining mathematical thinking look like in our classroom during math? *

- Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.
- Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.
- Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.
- Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.

Who engages in creating, demonstrating, or showing mathematical representations (visuals or models)? *

- Representations are missing, or teacher shows them to students.
- Students learn to create math drawings to depict their mathematical thinking.
- Students label their math drawings so that others are able to follow their mathematical thinking.
- Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.

How would you describe building student responsibility within the community, with regard to math? *

- Culture supports students keeping ideas to themselves or just providing answers when asked.
- Students believe that their ideas are acceptable by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
- Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
- Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Appendix B
Student-generated Observational Data Form: Tally Marks and Reflections

<table>
<tr><td>Student Name:

Work:</td><td colspan="2">Date:</td></tr>
<tr><td colspan="3">Partner(s):</td></tr>
<tr><td colspan="3">Sentence Starters for reflection(s) on today's Math work using discourse (Red Cards)</td></tr>
<tr><td colspan="2">I started solving the problem by…</td><td></td></tr>
<tr><td colspan="2">The strategy that makes the most sense to me is…</td><td></td></tr>
<tr><td colspan="2">A place where I got stuck was…</td><td></td></tr>
<tr><td colspan="2">I need help understanding…</td><td></td></tr>
<tr><td colspan="2">One thing I like about my strategy is…</td><td></td></tr>
<tr><td colspan="2">One thing I like about my partner's strategy is…</td><td></td></tr>
<tr><td colspan="2">Something new that I learned today was…</td><td></td></tr>
<tr><td colspan="2">I still am not sure about…</td><td></td></tr>
<tr><td colspan="2">I noticed a connection between…</td><td></td></tr>
<tr><td colspan="2">Something that is important to remember is…</td><td></td></tr>
<tr><td colspan="2">I was really surprised when…</td><td></td></tr>
<tr><td colspan="2">This is similar to…</td><td></td></tr>
<tr><td colspan="2">I agree with…</td><td></td></tr>
<tr><td colspan="2">I disagree with…</td><td></td></tr>
</table>

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Appendix C
Teacher Tally Log of Discourse Strategies Use

<table>
<tr><td colspan="6">Tally Chart Observation Tool for Blue Cards</td></tr>
<tr><td>Day and Date of Observation:</td><td colspan="5">M T W TH F</td></tr>
<tr><td>Number of Children Present:</td><td colspan="5"></td></tr>
<tr><td>Tally Recorded by:</td><td colspan="3">Circle: Teacher or Student Name:</td><td colspan="2">Grade:</td></tr>
<tr><td>Weather:</td><td colspan="5"></td></tr>
<tr><td>Special Circumstances:</td><td colspan="5"></td></tr>
<tr><td>Tally</td><td>Time</td><td>Discourse Strategy Used</td><td>Name of Math Lesson/ Work</td><td>Lesson = L
Work = W</td><td>By Teacher = T
By Student = S</td></tr>
<tr><td></td><td></td><td>Does your partner's strategy make sense?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Can you draw a picture or make a model to show how to solve the problem?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>How did you get your answer?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Do you want to revise your strategy or answer?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>How can you be sure your answer is right?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>How did you begin to think about this problem?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>What is another way you could solve this problem?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>How is your strategy different from or the same as another strategy?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Break the problem into parts. What would the parts be?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>What part of another person's solution do you want them to explain more specifically?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Does that strategy always work?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Can you think of a case in which that strategy wouldn't work?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>How did you organize your information? Your thinking?</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Was your group participation appropriate and helpful?</td><td></td><td></td><td></td></tr>
</table>

		Do you see any patterns?			
		Where could you get more information?			
		How would you check your steps or your answers?			
		What did not work?			
		How is your solution method the same as or different from another student's method?			
		Other than retracing your steps, how can you determine if your answers are appropriate?			
		How did you organize the information?			
		How could you solve this using tables, lists, pictures, or diagrams?			
		What ways have you tried? What steps did you take?			
		How would your solution look if you used another model?			
		How would you draw a diagram or make a sketch to solve the problem?			
		Is there another possible answer?			
		Is there another way to solve the problem?			
		Is there another model you could use to solve the problem?			
		Is there anything you may have forgotten?			
		What was you estimate or prediction?			
		How did you think about the problem?			
		How confident are you in your answer?			
		Is the solution reasonable, considering the context?			
		What patterns to do you see?			
		What picture could you draw to show the problem?			
		What strategy did you use?			
		Explain your partner's solution to them. Are there any steps you need to ask about?			