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Improving Assessment Outcomes in Algebra and Functions Through Concrete Materials and Direct Instruction

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Abstract

Critical thinking consists of the mental processes, strategies, and representations individuals use to solve problems, make decisions, and learn new concepts. We observed our 6th-year students struggling with algebra functions and problem-solving. These observations motivated us to engage in active research to improve the students' critical thinking and problem-solving skills. We administered interventions that supported the language transfer between traditional mathematical language and Montessori mathematical terms. We utilized direct instruction with concrete algebra materials to lead the students to abstract work in algebra. We also used formal assessments and observations to decide what lessons the students should get next. Our explicit instruction included Hands-On Equations created by Borenson to provide students with a concrete representation of algebraic skills and guide them to abstract understanding. This action research occurred within an Upper Elementary Montessori classroom and included seven 6th-year students aged 11 to 12. Two Montessori co-teachers conducted the study within eight weeks during January and February of 2022. Overall, students showed improvement on assessments, enhanced interest in math, and increased confidence when utilizing their problem-solving skills on problems involving newly learned concepts.

Keywords: Hands-On Equations, algebraic thinking, problem-solving, critical thinking, algebra, and functions

Worldwide, countries seek to improve problem-solving skills and critical thinking skills in their public education, including here in the United States (Ni et al., 2018). Problem-solving skills and critical thinking are essential to a student's education and to life after school. These skills are also essential components of navigating mathematics. Over the years, our school has worked on improving its mathematics curriculum. As we investigated the finer points of improvement, our upper elementary team agreed that real-world application and problem-solving skills were the weakest points for our students. Data from the Northwest Evaluation Association (NWEA) assessments, administered annually, confirmed our theory.

In September of 2022, our 6th-years took their annual NWEA standardized tests in mathematics and reading. These tests allow teachers to help in several areas; to gauge where students are in their math and reading skills, map out their expected progress, highlight areas they can continue to practice, and what lessons could come next. Based on data for the 6th year students, on average, they scored 3% lower in algebraic functions than in other tested areas.

We wanted to improve students' algebraic and critical thinking skills and problem-solving skills for our action research study. We taught them how to work through a problem step by step. They learned to list important information, draw diagrams or pictorial representations based on relevant information, and list their steps to solve their problem. We also directly instructed students about cross-curricular connections and how to apply past knowledge to present problems.

We introduced language that helped students identify when they utilized algebraic thinking skills. This identification improved their confidence when they came across vocabulary terms such as expression, variables, or evaluate. They also learned how to apply that language to visual representations. Students discovered that their flexibility in thinking and solving problems improved when they thought through information and skills they already possessed. For example, their lessons in Geometry introduced them to writing algebraic expressions, and their lessons in Physical Science have introduced them to variables.

Furthermore, we gave lessons to students using Hands-On Equations. Hands-On Equations served as the concrete work that introduced students to algebraic functions and allowed them to practice sensorially. The Hands-On Equations system showed students how and why expressions are balanced. Overall, we hoped to improve their algebraic and critical thinking skills and their abilities to apply algebraic functions in their work and on their assessments.

Theoretical Framework

It is important to address missing variables that prevent students from bridging the gap between concrete algebraic thinking and abstract algebraic thinking. Kolb's experiential learning theory states that experiential learning is the best method to understand the world. Learning about the world happens through hands-on experiences and interactions (McAuliffe & Eriksen, 2010). Kolb's theory aligns well with Montessori philosophy and the use of the Hands-on Equations materials.

Kolb's learning theory connects to our primary focus, which is bridging a concrete understanding to a conceptual understanding of algebraic thinking in mathematics (McAuliffe & Eriksen, 2010). Our second focus derives from the Montessori philosophy that concrete and abstract learning happens between the child and the materials. We chose to implement Hands-On Equations created by Borenson and the opportunities to work on real-world applications. McCoy (2016) states that experiential learning improves critical thinking skills. Additionally, experiential learning offers a more engaging route to learning complex mathematical concepts (Novak & Schwan, 2021).

Literature Review

At our school, the 6th year students' test scores are three percent lower in algebra and functions than in other math content areas in the Northwest Evaluation Association (NWEA) assessment. We believe the data correlates with a lack of materials in the classroom to navigate from concrete to abstract understanding in the given mathematical area, a lack of confidence in mathematical applications, and unfamiliarity with problem-solving procedures. David Kolb's theory includes four learning cycles: concrete learning, reflective observation, abstract conceptualization, and active experimentation (McAuliffe & Eriksen, 2010; West Governors University, 2020). We plan to provide our students with experiences using Hands-on Equations, direct instruction for solving problems, teaching them to make connections from past lessons to new lessons, and analyzing their work and learning through the lens of experiential learning theory. We will stay within the bounds of Montessori philosophy.

Pedagogy

It is crucial for students to understand why mathematical procedures work rather than memorizing procedures (Pennsylvania Department of Education, 2017). Flores et al. (2014) defined conceptual knowledge as "understanding the fundamental tenets of a domain, as well as the connections between concepts within a domain." (p. 75). They also defined procedural knowledge as "the ability to execute a sequence of mathematical manipulations to solve a problem." (Flores et al., 2014, p. 75). Concrete-Representational-Abstract (C.R.A.) is a method used to teach mathematics and connect procedural knowledge to conceptual knowledge (Mudaly

& Naidoo, 2015). C.R.A. and the Montessori method have several similarities. C.R.A. requires "Stages of Representation," which are analogous to Montessori's Three Period lessons.

The learning pathways of C.R.A. are learning through movement, more abstract representations, and abstract symbols. In Montessori classrooms, new concepts are introduced using physical representations through materials designed by Dr. Maria Montessori. Al Sayyed Obaid (2013) found that when students moved from concrete to abstract thinking, using materials and movement helped them visualize the steps in mathematical algorithms. Manipulating materials offers many opportunities and neural pathways to solidify a concept into memory, including visual, tactile, and kinesthetic (Al Sayyed Obaid, 2013). Students use concrete materials to derive *why* procedures work, reinforcing their conceptual understanding.

When students make multiple representational connections, they better understand why their approach is successful. Montessori classrooms echo this practice when students move to more abstract materials, such as the test tubes for division. Students can also draw pictorial representations of materials or pertinent information from the problem. This understanding allows students to generalize and make connections between different approaches and problems, apply their understanding to working out new problems, and solidify their abstract understanding of a concept (Ni et al., 2018). When students display that they have gained an abstract understanding of a concept, practicing application is vital to ensure they make connections to previously learned concepts.

Real-world applications are included in our classroom as a follow-up so students can practice applying their understanding and make their work relevant. Once students begin to view math as conceptual rather than only procedural, students' interest and confidence strengthen (Ni et al., 2018). Students often view math as a procedure or a series of algorithms to memorize and do not see value or purpose in their work, even when solving real-world problems (Ni et al., 2018). When students learn concepts and different paths, strategies, and processes, students' views on math improve, interest in math and class participation increases, and students agree that math is purposeful work (Ni et al., 2018).

Richland et al. (2012) observed that students need to view mathematics as a goal-oriented structured system. Students need to understand and acknowledge that they can reason through problems using logic. The end goal is for students to understand *why* concepts work the way they do. Teachers strive to provide lessons that teach why concepts work and for students to commit lessons to long-term memory (Richland et al., 2012). In contrast, students try to memorize procedures well enough to pass an assessment using short-term memory.

Most students view math as a set of rules to memorize. If these rules are not memorized, it prevents them from attempting more challenging problems (Ni et al., 2018; Walick & Burns, 2017). That mentality creates a stigma that they are not good at math and that math is not purposeful work; this translates into a lack of effort from the child to move from concrete to conceptual understanding. Educators noted the need to break the mold that leads to the misconception that math only consists of rules and procedures that need to be memorized. Once students master foundational skills, teachers can teach them to use critical thinking to navigate more complex math experiences. The necessity for a proper and solid understanding of the foundational math skills should be underscored (Richland et al., 2012; Lekwa et al., 2019).

Ni and associates (2018) investigated the relationship between cognitive features of mathematical instruction tasks and student learning outcomes among thirty fifth-grade classrooms in China. These classrooms used a new mathematical curriculum developed by China's Department of Education (Ni et al., 2018). This study asked if the changes in how mathematics is taught in China improved students' views on their mathematical abilities and mathematics in general. Improving mathematical acumen is an issue that many countries, including the United States, are currently focused on (Mullis et al., 2020).

Chinese students generally excel with accuracy, efficiency, and endurance when figuring out math problems, which Ni and associates (2018) credit to the mathematics curriculum that emphasizes the “fundamental knowledge and skills” (p. 704) required to be proficient in the areas of mathematics and problem-solving. They also acknowledge that skilled and experienced teachers are integral to the success of a written curriculum (Ni et al., 2018). They considered two aspects of teachers’ practices within the classroom; how they implemented different steps of instruction and how the teacher interacted with the students during the lesson.

Many students within the Beghetto and Baxter (2012) study perceived that mathematical knowledge came from external resources, such as textbooks and teachers. Students must learn that they can be resources to find a way to solve a problem. Relying on teachers and textbooks as the only resources to find the solutions could limit students' regard for their own abilities. If students work under the assumption that there is only one way to get an answer, they often misjudge their abilities and avoid attempting to solve problems because they think they do not know how to do it. This misconception creates a false view that they are not mathematically talented. Then students begin to view incorrectness as failing rather than a learning opportunity (Beghetto & Baxter, 2012).

Students' personal beliefs about their abilities in mathematics also influence their views on whether they can understand math and how it works (Beghetto & Baxter, 2012). Beghetto and Baxter (2012) explored students’ beliefs about their own abilities and understanding of mathematics and how they were connected to the way math was taught. Students look to their teachers as the primary source of feedback to check for understanding of lessons, materials, and work (Beghetto & Baxter, 2012). Positive beliefs and understandings occurred when teacher feedback was constructive and guidance, rather than criticism, was given (Mesler et al., 2021). When teachers judge student academic performance and are perceived to have a fixed mindset, negative feelings become associated with interactions. Consequently, students' perceptions of math may become more negative (Beghetto & Baxter, 2012; Mesler et al., 2021).

Brown (2011) noted that instructors teach algebra in a lecture format in a typical classroom setting. Mathematical concepts that require high function reasoning can be too complex for students of varying abilities to grasp when only taught through one mode (Al Sayyed Obaid, 2013). Accommodations and differentiation are essential tools to reach children of varying abilities. Teachers are primarily responsible for differentiating and teaching to all of their students' abilities.

Concrete materials found in Montessori classrooms and systems like Hands-On Equations allow learning to be differentiated and paced by the student. The materials also engage many senses to help children understand these concepts. Al Sayyed Obaid (2013) found that 6^{th}year students with learning disabilities improved their scores significantly compared to the control group when taught using a multisensory approach. Learning differences and disabilities regarding students' learning are important aspects of how educators present lessons.

Between 2019 and 2020, more than seven million students received services under the Individuals with Disabilities Education Act (IDEA). The United States scored in the top quarter in 4^{th} and 8^{th} grades on international tests, but no other country had a more significant gap between the lowest and highest performing students (Mullis et al., 2020). Students who had low performances in math often struggled with computation and problem-solving. In order to give students skills to reason through math problems, it is necessary to improve their computations and problem-solving skills (Brown, 2011).

Scaffolding mathematics lessons, along with guidance and support from the teacher, is an effective method to reach students with varying mathematical proficiencies (Richland et al., 2012). Once children reach abstraction, practicing real-world problems allows students to apply their learning. It is crucial that teachers guide students to make connections between lessons, materials, and abstract work. Connections help ensure that work feels purposeful while also creating context. These connections allow students to successfully transfer understanding between mathematical concepts (Richland et al., 2012).

When teachers take away students' productive struggle, the meaning of the work is lost (Richland et al., 2012). Productive struggle allows students to navigate problems using logic and prior knowledge. Students must attempt different courses of action when they struggle and find the meaningful connections that transfer between old concepts and new ones. It is imperative that students learn that mistakes are part of the learning process and signals when a different strategy might be necessary, rather than believing they have failed. This paradigm shift happens when the focus includes how concepts work and encourages students and teachers to understand why concepts work (Richland et al., 2012). Teachers can provide multiple representations that allow different connections to develop (Laski et al., 2015). If students understand mathematical procedures and concepts, educators can foster deep and flexible mathematical awareness in their students by working with real-world problems and applications (Richland et al., 2012).

However, making connections and comparisons is not helpful if students do not transfer their prior knowledge in order to solve a new problem. Richland and associates (2012) found that only fifteen percent of students in their study attempted to reason through given problems. Most students stopped working through the problem when they realized they did not have the rules or procedures memorized. It is difficult for students to memorize all the rules, formulas, and algorithms in mathematics. Generalization between concepts and problem solving using rational reasoning happened when students and teachers worked together on making connections and building strategies that allowed children to piece together prior knowledge with the new concept (Richland et al., 2012).

Beghetto and Baxter (2012) found that teachers should guide students through various ways to solve a problem to increase their innovative ideas and willingness to take risks and decrease their fear of being wrong. Teachers need to be fluent in mathematical concepts, not just procedures, to teach children how to make connections between these multiple pathways, the basic skills they already know, and more complicated problems that can be deduced logically (Ni et al., 2018; Cai, 1998). For example, if a child struggles to memorize the algorithms of long multiplication, that child can find the answer if they understand what is literally happening in the problem. If students understand that each hierarchy of the multiplicand is multiplied by the multiplier, then the problem can be solved with logical deduction without knowing an algorithm. An example of teaching this concept in the Montessori classroom is the multiplication checkerboard.

Concrete to Abstract Thinking

It is critical that educators ensure that students understand foundational mathematical areas such as operations, math facts, multiples, and divisors. This understanding is essential for students to use reasoning by comparison (Richland et al., 2012). If a student is not fluent in the foundational mathematics concepts with a concrete understanding of why they work, it becomes too difficult to recognize analogies between various problems (Goswami, 2002). For example, teachers cannot expect students to succeed with long division work if they have not mastered multiplication facts. Richland et al. (2012) point out that once students have these necessary skills, they can tackle more complex problems.

> Once students achieve mastery of necessary skills, however, they will gradually be able to handle more cognitive load. They may benefit from more effortful work to align and map between source and target analogs. Thus, the role of teacher support for relational thinking and sense-making may shift throughout students' learning (p. 200).

Richland et al. (2012) studied students who embraced the struggle that comes with facing unfamiliar problems in mathematics. Richland and associates (2012) defined struggle as the duration of effort students employ "to make sense of mathematics, to figure out something that is not immediately apparent" (p. 387). A student's prior knowledge affects their level of struggle. There should be a healthy balance, so the requirement to reason out the problem causes a struggle while finding a solution remains attainable.

If a student thinks they are deficient in a specific math skill that allows other students to succeed, it can have real-life effects on their performance even if they possess the required skills (Brown, 2011). Students become stuck in the "struggle period" for longer than necessary, which can cause frustration and discourage working on less familiar work (Beghetto & Baxter, 2012). Suppose a child is allowed to struggle for too long without any productivity. In that case, they feel they have an inadequate foundational understanding to reason through many of the mathematical problems (Richland et al., 2012). Support from teachers to bolster students' understanding of what is happening with concrete materials is vital to solidifying a child's path to abstract understanding (Brown, 2011; Ni et al., 2018).

Raymond and Leinenbach (2000) discovered that teachers who used textbooks exclusively were skeptical of hands-on or experiential teaching strategies. Once teachers presented the materials to students, both teachers and students had the opportunity to understand *why* concepts worked through a concrete approach that the textbooks could not provide. Students practiced utilizing the hands-on materials and, at the same time, asked themselves why the materials navigated them to their solution each time (Raymond & Leinenbach, 2000). Teaching with concrete materials provides a differentiation that helps fill gaps in students' understanding of concepts. Students use materials to analyze and rationalize why concepts work. Working through the materials multiple times over an extended period is beneficial to most students (Laski et al., 2015).

Al Sayyed Obaid (2013) wrote that schools attempted to create a concrete understanding of algebra through computer simulations. It was a concern that the drawbacks of screen time could outweigh the benefits of the simulations (Al Sayyed Obaid, 2013). Her research found that simulated and real manipulatives created positive attitudes towards math. Students learned to trust the process when using materials (Raymond & Leinenbach, 2000). This phenomenon is observed in Montessori classrooms as well. We have also observed that students’ confidence improves when they realize they can navigate to the solution with materials.

While using the C.R.A. protocol, children move from concrete to visualization and retain a mental image of the materials used in the concrete portion. This visualization is necessary for problem-solving (Borenson, 1987). Repetitive practice is necessary to navigate through the three steps of C.R.A. More abstract visuals such as graphs, geometric shapes, diagrams, and videos are meaningful in the representational period (Mudaly & Naidoo, 2015). Children move to abstraction once they model critical concepts at the representational level. Instruction from the teacher during the introduction of visuals is necessary rather than expecting students to infer what the materials represent (Richland et al., 2012). Montessorians often say the materials speak for themselves. While we agree with this, for the most part, it is essential to verbalize specific language when closing the gap between concrete and abstract understanding.

Mudaly and Naidoo (2015) advocate using C.R.A. in classrooms in South Africa. As previously mentioned, C.R.A. is a three-step process that focuses on bridging the gap between concrete and abstract understanding of mathematical concepts. Mudaly and Naidoo (2015) wrote that teachers mistook students' lack of confidence with abstraction as unpreparedness and uncooperativeness. Mudaly and Naidoo (2015) point to the lack of skills on the teachers' part to support their students through the path from concrete to abstract understanding. Students must master each step before moving on to the next: concrete, representational, and abstraction. Mudaly and Naidoo (2015) found that teachers who experienced success with students' understanding of abstract mathematical concepts used objects, hands-on learning, and engaged in meaningful discourse with students to achieve better student understanding.

The Hands-On Equations Program is a concrete material used to represent algebra functions. Students use different color pawns and number cubes to represent the variables and numbers that make up an algebraic equation. Within the program, teachers taught students how to make a "legal move" (Borenson, 1987). The term legal defined what moves a student could and could not move. The materials bridge the gap between concrete and abstraction. Raymond and Leinenbach (2000) conjectured that some students felt they needed the manipulatives to show what they knew. Close observation of the students will help teachers know when to guide them away from materials and towards abstraction.

Teacher guidance ensures that students do not depend upon the materials to avoid what they view as failing (Raymond & Leinenbach, 2000). The materials should be a step in moving from concrete understanding to abstraction. Once students are comfortable physically moving the materials to model a function, they begin drawing the manipulatives on paper instead of having the materials in front of them. Students drawing concrete materials show that they can visualize the materials even though the materials are not available. In the Pyyry et al. (2017) study, students reported visualizing the materials in their minds as they drew them to solve a function.

Critical Thinking

Beghetto and Baxter (2012) explored critical thinking through student beliefs and understanding of elementary science and mathematics. The increased effort necessary for critical thinking causes children to gravitate towards other academic areas and away from math. Persistence is an essential characteristic of critical thinking (Beghetto & Baxter, 2012). If students do not feel confident in their mathematical abilities, they cannot feel effective in that domain. Without practice and persistence or a sense of efficacy, students struggle to think through a problem critically (Beghetto & Baxter, 2012). In our classroom, students are encouraged to spend a minimum amount of time in each subject area; for example, the minimum time in math is forty-five minutes. The hope is that this gives students enough time to engage in productive struggle and deters them from quitting prematurely.

Research suggests that analogical reasoning is at the core of what is unique about human intelligence (Richland et al., 2012). Analogical reasoning happens when objects or ideas are compared, and how they are the same is used to come to a conclusion when solving a problem (Bartha, 2019). The fundamentals of analogical reasoning with causal relation appear as early as infancy in humans and perhaps in nonhuman animals (Bartha, 2019). Children's analogical ability improves as they develop, with older children focusing on more specific details between similarities and differences (Mayer et al., 2014; Bartha, 2019). Analogical reasoning allows a child to see similarities between a current problem and previously solved problems. This reasoning leads to the transfer of knowledge to the current problem and the use of the skills developed previously (Richland et al., 2012).

Students often need direct instruction from the teacher to practice identifying and using analogical reasoning (Richland et al., 2012; Cai, 1998). Teachers in the United States are "least likely to support their students in reasoning comparatively" (Richland et al., p. 198) to teachers in other countries. This lack of support from U.S. teachers means that even though they allow opportunities for analogic reasoning during lessons, the absence of support strategies leads to students who cannot utilize these opportunities and fail to notice or draw the relevant structural connections (Cai, 1998). Work is needed to educate teachers about implementing support strategies that show students how to generalize what they know in math and make connections from past solutions to draw conclusions for a new problem, thereby logically reasoning out a solution (Richland et al., 2012). Finding a balance between support and independence is important. In the Montessori classroom, careful observations of the students' work are necessary to find that balance.

Conclusion

Teaching mathematical concepts and procedures is essential to students' ability to think critically through math problems. Teachers face this challenging task worldwide (Mullis et al., 2020). This challenge is also faced in Montessori schools. However, Montessori schools are specially equipped with thoughtfully designed materials that bridge concrete understanding to abstract understanding in mathematics. Montessori math materials are scaffolded so that one concept builds upon previously mastered concepts and can be applied to new concepts. The research presented in this review shows that experiential learning improves critical thinking skills. Research also points out that when teachers help students connect concrete materials and abstract math concepts, they find math more interesting, and their confidence in their mathematical abilities may increase.

What Montessori seems to lack is the direct instruction component that assists students with bridging the gap between completing work with concrete materials and applying their abstract understanding. Evidence that the Montessori materials and other experiential learning materials support that bridge is largely anecdotal. We hope that our action research helps support this idea with the addition of empirical evidence.

Methodology

Basic Design of the Study and the Variables Examined

We began our research by acknowledging that the guide significantly encourages each child's confidence to succeed in mathematics in an upper elementary school classroom. Our students' data from the NWEA assessments showed that they needed more support in algebraic functions and critical thinking than in other areas. Students received direct instructions to show them that they already possessed the necessary background knowledge to complete their work successfully. We also taught them how to transfer those previously learned concepts to new concepts. We started by having conversations that helped relate new math skills to previously mastered content. When students began to connect concepts, they saw purpose in their work. They also gained confidence when faced with new challenges.

Montessori math lessons were taught once a week and focused on critical thinking and algebraic functions. Students practiced their new skills starting on Mondays after their math lessons throughout the week until Thursday at the end of the day. We used the Big Ideas Math textbooks for assignments. The textbook proved to be a beneficial resource for students. It included practice problems related to real-world situations, included an abundance of computation practice, and provided many opportunities to apply the skills within the context of story problems.

We implemented lessons from Hands-On Equations to our 6th years in two small groups on Thursdays. The Hands-On Equations lessons focused on working through algebraic equations by learning how to navigate the steps using concrete materials. On Fridays, we began the day with Problem Solving Friday. Students met with their math group and chose a table to work collectively. We assigned each group a story problem that required them to apply their understanding of the new skill presented in their lesson on Monday. Students had a set amount of time to highlight important information, followed by a prompt to solve the problem independently. We directed students to solve the problem collaboratively and present their work to the class at the end of the work session.

We took anecdotal notes throughout the weeks as they practiced and during Problem Solving Fridays to help guide the upcoming Monday math lessons. The 6th year students also helped us decide how to guide instruction by completing an anonymous four-point scale survey. The survey asked them to rate how they felt they could apply their understanding (see Appendix A). Finally, we collected and analyzed Problem Solving Friday work from each group to help determine their ability to apply what they learned before making any adjustments and preparing for the next week of lessons. We administered formative assessments from the Big Ideas math textbooks to check for understanding. The students checked their work with teacher supervision. This allowed students to assess their learning and get immediate feedback on their work.

Population

We focused our research on our 6th year students, who are typically 11 to 12 years old. Students who participated in the study ranged from nine to twelve years old. The research was conducted in an Upper Elementary Montessori classroom in Northern Indiana. There were three biological males, all of whom identified as male. There were four biological females; three identified as female, and one as non-binary.

Intervention

The action research was conducted between Monday, January 3rd, 2022, and Thursday, March 3rd, 2022. We scheduled Montessori math lessons with our two 6th year math groups on Mondays. The students planned when they would practice the new skill for the week. They set aside time each day, Monday through Thursday, to work on the concept introduced to them that week. We expected the students to use the materials introduced in the lesson until they demonstrated mastery. Big Ideas Math: Modeling Real Life textbooks for fifth, sixth, and seventh grade were the resources used for follow-up. We implemented Problem Solving Friday each Friday morning from 8:30 to 9:30. Students worked in their math groups to solve a story problem. We chose problems that required students to pull from previous knowledge and apply the new skill they learned from their Monday lesson.

We reminded students to connect past knowledge from previously mastered concepts and incorporate them into the work for the new concept. We reviewed some of the topics students had mastered and helped them see the connections with the current weekly lessons. These connections reminded them that they already possessed the necessary knowledge to succeed in their new work. The follow-up work overlapped mastered concepts with new concepts. This practice kept with the Montessori tenet of staying rooted in work students are familiar with while working on new and unfamiliar concepts.

Follow-up assignments came from the Big Ideas Math by Larson textbooks and were particularly useful when looking for real-world application problems. The work also challenged the students to think outside the box and stay mentally flexible when applying the rules and formulas they learned. The textbooks categorized word problems in several ways, with each category offering a variety of challenges. For example, Modeling Real Life and Digging Deeper pushed students to apply what they already knew and what they learned in the Monday lesson in different ways.

Students gathered into their math groups for the Problem Solving Friday sessions on Friday mornings. We gave each group a problem they had not worked on before. First, they highlighted information they saw as critical to solving the problem. Then, they attempted to solve the problem independently. While working independently, they used their notes, wrote down ideas and questions, and drew pictures of their process. Next, each student presented their problem-solving method to their group. The group then discussed and debated the best method or methods to find the correct solution to the problem. After that, groups worked together to solve the problem. The group chose a scribe to write the steps they took to solve the problem on a separate piece of paper and draw any necessary pictures. Finally, each group chose a representative to present their problem, the method used to solve it, and their solution to the problem to their classmates.

We utilized the Hands-On Equations system developed by Dr. Henry Borenson. He designed this system to show concrete representations of algebraic expressions for math learners. Dr. Borenson's balance model for solving equations uses the visual and kinesthetic instructional approach to simplify complex algebraic concepts. The hands-on and intuitive approach improved students' self-esteem and interest in mathematics. It was a game-like approach that intrigued students through legal moves used to solve the equations and reinforced the concepts at a deep kinesthetic level. The algebra concepts taught with Hands-on Equations included variables, evaluating expressions, the meaning of algebra equations and formulas, and balancing equations.

Each Thursday that we did not administer a formal assessment, our seven 6^{th} year students received direct instruction using the Hands-On Equations system. We split the students into two groups for the lessons, displaying pawns and number cubes on a balance. This visual showed students a concrete representation of an algebraic equation. Each lesson took the students one step further into complex algebraic equations. Our students were instantly intrigued and fascinated with the materials and the challenge of comprehending algebraic functions. The first two lessons instructed us to guide the students through examples. Students created the equations using their own set of manipulatives. The Hands-On Equation manual did not provide follow-up for students in the first level. However, we created follow-up for students to practice throughout the week because the students asked for follow-up work. They were eager to learn and practice with the new materials. After level one, the system provided practice for the students to complete throughout the week to prepare them for the next lesson.

Procedures Used to Gather Data

During our maths lessons, we tracked how students felt about the difficulty of their work through conversations and notes. We also made observations of students' level of participation during the lesson. Monday through Thursday, we continued to write anecdotal observations regarding students' math practice. We expected students to utilize Montessori materials until they demonstrated understanding and fluency. We considered a concept mastered when a student could explain the process of their work and its connection to the materials.

We assigned their follow-up from Big Ideas Math: Modeling Real Life textbook by Larson. Follow-up consisted of examples from the beginning of the chapter to help solidify concepts, concepts and skills practice problems, and word problems from the various categories. The Big Ideas Math texts have several categories of word problems: Structure, Modeling Real Life, You Be the Teacher, Dig Deeper, Reasoning, and Number Sense. We assigned various problems and noted why their answers were correct or incorrect. We observed their work progress throughout the week and noted if and why they asked for help. We looked for understanding rather than just correct or incorrect answers when we checked their work. We used any incorrect answer caused by a lack of understanding as a guide to show where the student needed support. If the error resulted from miscalculations, we noted which operations they needed to practice.

During Problem Solving Friday, we walked around and observed throughout the session. We wrote observations about student participation, their contributions to the discussion, their questions, and how they explained their ideas. The students did not check for correct answers because this work focused on the problem-solving process, not the solution. We gathered their work at the end and checked for understanding and possible errors. We checked work for correct and incorrect answers and deciphered why answers were incorrect. After the Problem-Solving Friday session, we handed out short surveys to the 6^{th} years that asked them to anonymously rate how well they understood their work on a scale of 1-4. This short survey was anonymous to help ensure students' comfort so they could be honest and transparent about their perspectives (see Appendix A).

Hands-On Equations follow-up provided students with algebraic equations and gave them a concrete way to identify the variable's value and how to balance equations. They used a laminated picture of a balance to lay their materials on. The materials gave them a visual representation of the equation before computing what the variable represents mathematically.

In order to gauge whether students comprehended and retained the information they learned from lessons, we administered a pre-assessment, two benchmark assessments, and a post-assessment. The assessments contained the skills introduced during the Monday morning lessons. The assigned problems asked students to apply their understanding of the skills in story problems and algebraic functions. Students took these tests on Thursdays in place of Hands-On Equations lessons on January 3rd and 20th, February 17th and 21st, respectively. The students' scores were assessed based on whether the answer was correct or incorrect, and we used the assessments to plan future lessons.

Data Analysis

Data collection for our study took place in January and February of 2022. The first three weeks of data collection were affected by teacher absences due to each teacher testing positive for COVID and taking the mandatory ten-day quarantine. The absences caused us to miss a week, which necessitated an additional week to compensate for that lost time.

Mini-Lesson Follow Up

We taught math lessons during Monday morning work cycles. At the beginning of the study, the 6th year students were split into two groups according to their work pace for mathematical topics. Every week we collected data, focusing on whether children mastered the topic taught. We observed one group meeting mastery each week. The second group displayed inconsistent practice on their work, making it challenging to gauge mastery throughout the four days of practicing before application on Fridays. In hindsight, it was not realistic to look for mastery in such a short time. Therefore, we did not include a record of their mastery on follow-up from their weekly mini-lessons as part of the data collected. It would be more efficient to look for mastery throughout the school year as students practice and cycle back to concepts.

In the first week of the study, students could not apply the new skills learned on Monday to their Problem-Solving Friday lesson. We called the class to the community rug and asked the students to reflect on why they could not apply their understanding. Students admitted they did not complete their follow-up, and they were unable to apply their understanding due to a lack of practice throughout the week. In weeks two through eight, we observed students increasingly prioritizing math on their work plans throughout the eight weeks.

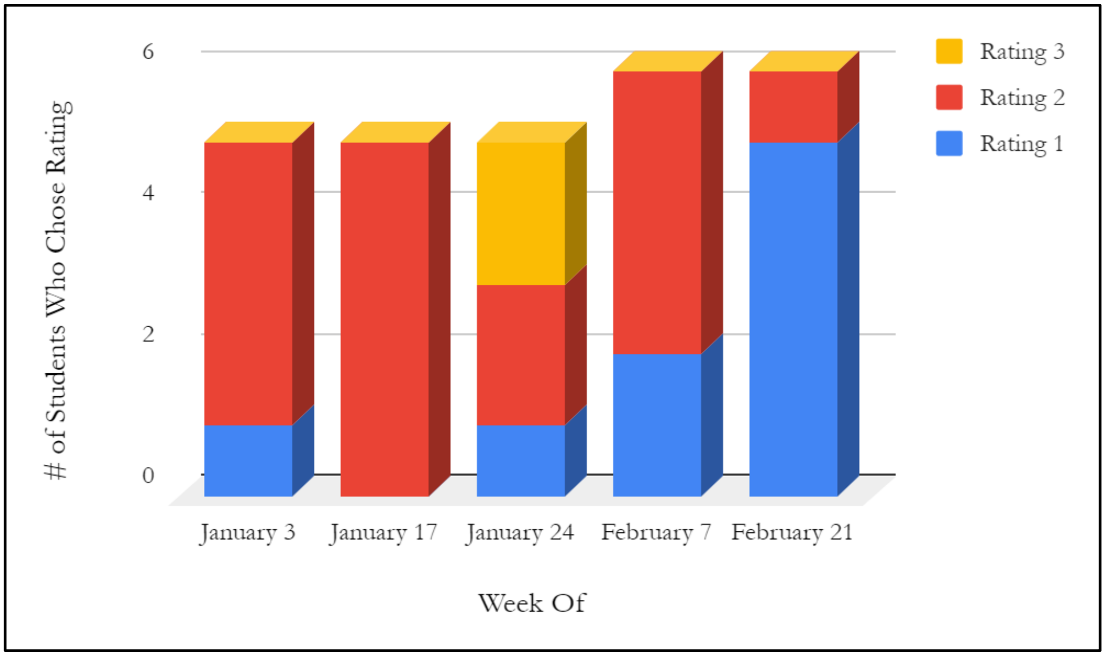
On week seven, the school was closed due to a scheduled Mid-Winter break. Therefore, we scheduled and gave lessons on Tuesday. Then, snow and ice caused the school to cancel on Friday. Students practiced their new skills two days that week and did not have the opportunity to apply their understanding to Problem Solving Friday. One of the math groups split into two smaller groups on week three due to a shift in ability levels. One student moved into seventh-grade math work while the other two students remained working at the end of the year sixth-grade level. The other group remained together with four students working on sixth-grade level work.

Problem Solving Friday

After students completed Problem-Solving Friday, the 6th year students completed an anonymous survey to analyze the depth of their understanding for the current week. Figure 1 shows that students were confident when applying the lessons taught to them. We initially thought that this would not be the case. There could be several factors contributing to this or a mix of factors. For example, we focus on positive self-talk as part of our Care for Self work in our classroom procedures. Also, the 6th years have been in the same classroom for two and a half years. The self-assessment asked each student to circle the number representing their feelings after the Problem Solving Friday work. They completed these assessments separately and anonymously and turned them in (see Appendix A).

Figure 1

Students' Self-Assessment Scores



Note. Figure 1 displays the self-assessment scores students gave themselves after completing their Problem-Solving Friday work. Ratings were numbered 1 through 4, 1 showing the most confidence and 4 showing the least confidence (see Appendix A).

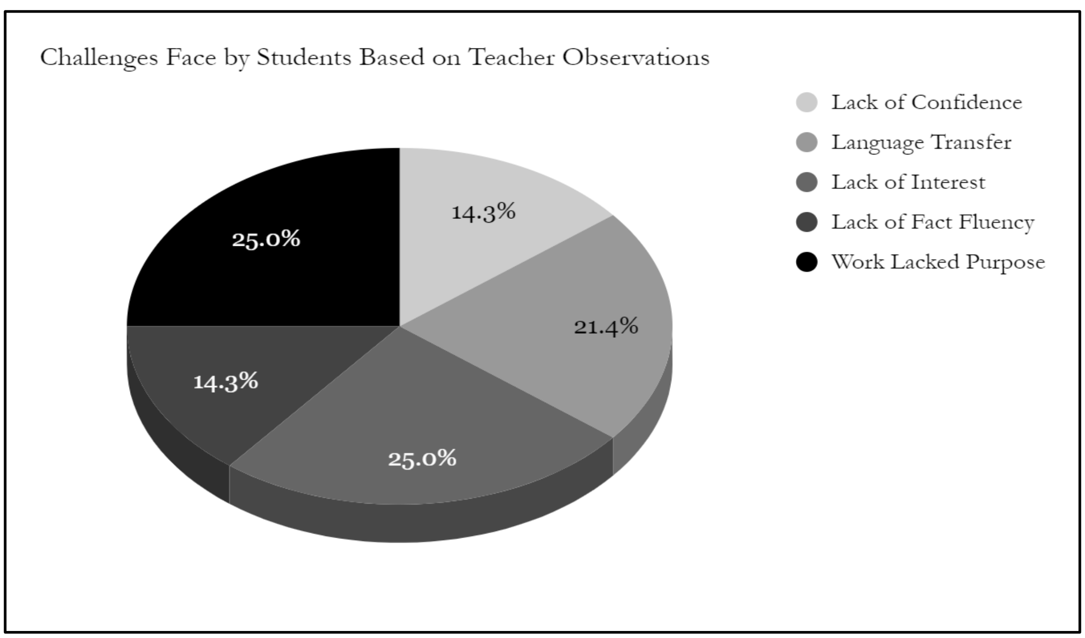
Unfortunately, our data collection was interrupted for three out of eight weeks. For the first three weeks of January, we both contracted COVID and therefore had to quarantine for ten days each. Our absence impacted the week of January 10th. Additionally, winter weather impacted the weeks of January 31st and February 14th, causing school closures. Therefore, we did not collect self-assessment data for those weeks. Due to illnesses among the students, we also had a different 6th year absence on January 3rd, 17th, and 24th. That left 5 data points for each of those weeks.

Observations

Our observation notes found five common challenges our 6th years faced. We discovered these themes by highlighting common language that we both wrote in our observations of students during lessons, work cycle, and Problem Solving Friday. Then we tallied how many times each showed up in our notes by limiting the phrases to what is shown on the graph in Figure 2; challenges with language transfer, lack of math fact fluency, confidence, interest, and purpose in math work. We did find that we were incorrect about the lack of confidence in at least three of our students. The self-assessments do not show a lack of confidence, and in that case, our data do not match.

Figure 2

Challenges Faced by Students



Note. Figure 2 reflects the challenges faced by our students. The percentages displayed reflect information found in teachers' observation notes.

The aspect of language transfer is a common problem with Montessori students. For example, phrases such as *evaluating the expression* rather than *solv*ing, or *place-value* instead of *hierarchy*, were words our students had to become familiar with. The language used in the Montessori lessons does not always transfer well to more traditional teaching formats such as textbooks and assessments. Once the students became familiar with various phrases, their comfort with the textbooks increased.

Math fact fluency is essential to completing higher-level thinking mathematical problems. If a student spends too much of their mental energy calculating simple facts or cannot detect early mistakes, it is difficult for them to track their work, stay with the problem, and get the correct answer (Baker & Cuevas, 2018).

Further observations caused us to note that we have three students who do not lack confidence but might be overconfident. Our observations showed us that their overconfidence led to disagreements in their groups, not listening to their group members' input, making simple calculation errors, and not checking their work. Overconfidence also showed up in students' work. It manifested as working quickly through calculations resulting in an incorrect answer, skimming the work rather than trying to gain a deep understanding, and not recalling details from lessons.

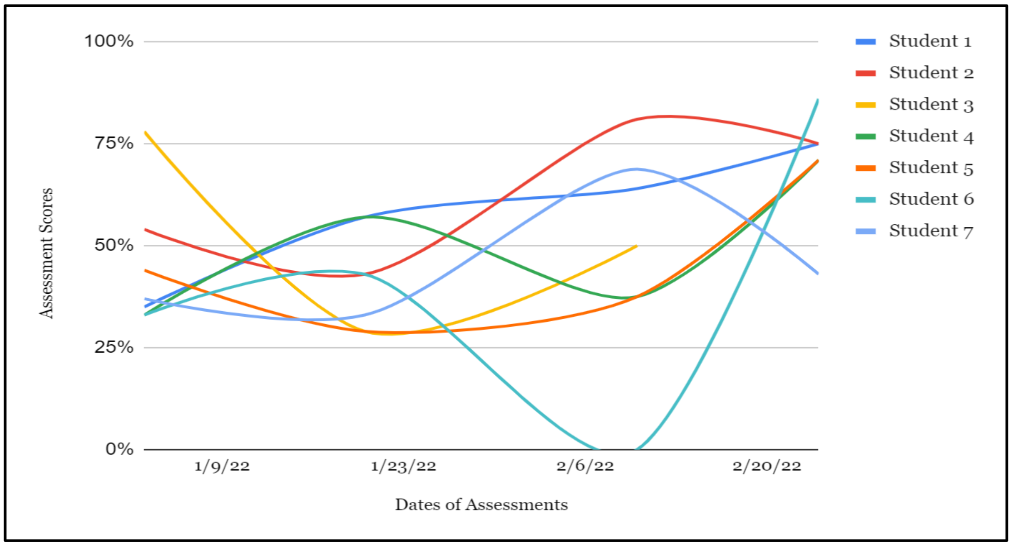
Two challenges that faced the students went hand-in-hand. Students did not see purpose in their work which contributed to a lack of interest in math. We observed both of these challenges in all of our 6^{th} year students. We observed an increase in interest and purpose when we implemented Problem Solving Fridays. The students regularly voiced their enjoyment in math lessons, Hands-On Equations lessons, and working together to solve their problems on Friday mornings.

Assessments and Benchmarks

We used assessments from the Larson Textbook to collect quantitative data throughout the eight weeks. We used the pre-assessment, benchmarks 1 and 2, and post-assessment from the fifth, sixth, and seventh-grade texts. Four of our students worked from the fifth-grade text, two worked from the sixth-grade text, and one worked from the seventh-grade text. We highlighted eight to eleven problems that contained algebra and functions and story problems within the assessment. We asked the students to complete the highlighted problems on the assessments on Thursday in place of their Hands-On Equations lesson. Students completed their pre-assessment on Monday, January 3rd, which served as a baseline. Then they completed benchmark assessments on January 20th and February 20th, and the post-assessment on February 24th. The post-assessment was an overview of concepts from previous lessons. Figure 3 shows each student's progress throughout January and February.

Figure 3

Formal Assessment Data



Note: Figure 3 displays how each student scored on the formal assessments throughout the study. Students completed a formal assessment on January 3rd, January 20th, February 10th, and February 24th. Student 6 completed her assessment but received a 0 due to calculation errors on all of her work. Student 3 was absent the day of and after the post-assessment and did not have a score for that assessment.

Students completed their formal assessments independently. The assessments helped us analyze and alter the weekly lessons given. Due to the requirement of independence during assessments, we were able to see that students five and six relied heavily on their partnership when completing their follow-up. We asked those individuals not to work together when completing math work. Student six received a zero percent score on her Benchmark 2 because she made minor calculation errors. She was able to navigate through much of the process independently but made mistakes with her math facts and how to label her final answer. Student three displayed a strong sense of mastery in the pre-assessment but was inconsistent and regressed on Benchmark 1 due to too much socializing and a lack of practice. Student seven avoided math at the beginning of the study. Once he consistently practiced his math skills, he made substantial progress shown on Benchmark 2.

The post-assessment gave us insight into individual student growth throughout the eight weeks. Student one made steady progress throughout the study. He included visuals and his work on the post-assessment, a practice carried over from Problem Solving Friday. Student two showed all of his work on the post-assessment to the degree of clearly depicting his thinking processes for each problem. His answer did not match his work for one of the problems, but we pointed it out to him and gave him credit for finding the correct answer. Student three was absent for the post-assessment and the day before and after. These absences caused her to miss the window of our study completion. Unfortunately, that means we could not include her post-assessment data in the study.

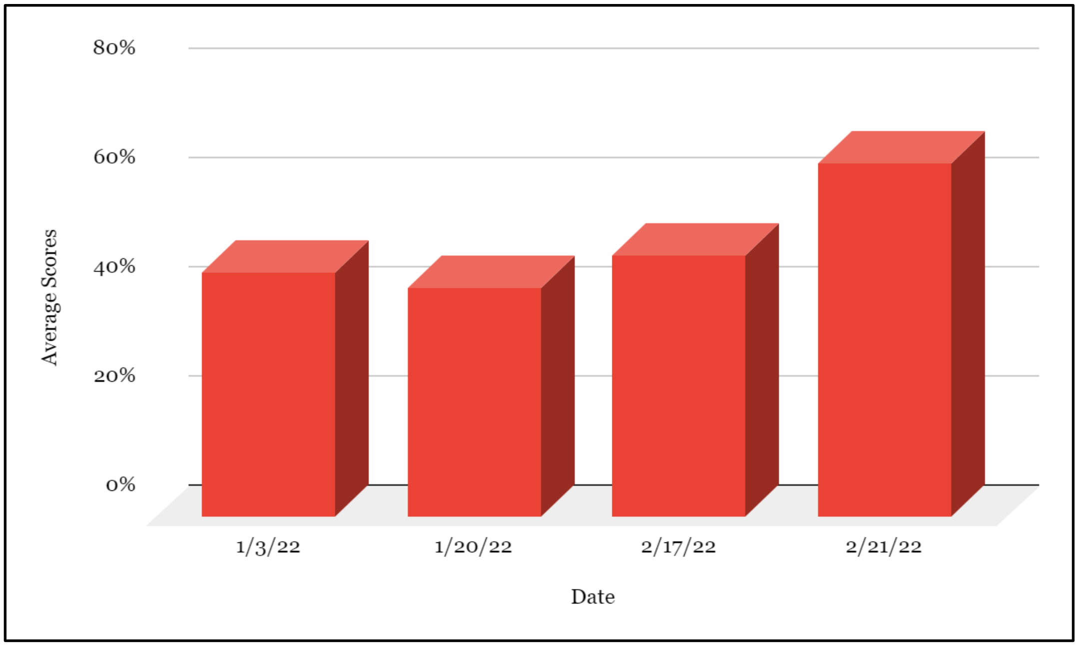
Student four completed their post-assessment during a tutoring session after school. They were observed abstracting in their head and showed minimal work on their assessment. We believe they made some errors they may have avoided if they had chosen to write their thinking on paper. Student five reflected a newfound sense of confidence in her math work. Her math fact fluency was accurate, showing a difference from her work on the pre-assessment eight weeks prior. She worked steadily, reflecting that she felt the work was purposeful, which is the opposite of the rushed demeanor demonstrated during the pre-assessment and benchmark one.

Student six showed her work, and much of the process was correct, but the numbers she pulled from the problem were incorrect. There was a disconnect between understanding the question asked and applying the correct process and numbers to find the solution. Student seven demonstrated some growth but not as much growth as expected. His inability to apply his understanding on the post-assessment reflected practicing for completion instead of preparing for the application.

Figure 4 shows general improvement in assessment scores and allowed us to conjecture how the students felt about their abilities in mathematics. The unexpected results of having a few students who do not struggle with their math work struggle to apply their learning was eye-opening. We frequently remind students that daily practice and checking for understanding are necessary for success in any area of academics. It shows that when students practice consistently and use concrete materials, their understanding may improve instead of simply memorizing mathematical processes and rules (Ni et al., 2018). Mathematics is a language, and without practice and application, its meaning is lost.

Figure 4

Average Math Assessment Scores



Note. Figure 4 displays the average math assessment scores from the formal assessments.

Conclusion and Recommendations

We addressed the missing components that helped bridge the gap between students' concrete algebraic thinking and abstract algebraic thinking. On the NWEA standardized assessment, the 6th year students scored, on average, 3% lower in Algebra and Functions than they scored in other areas of the math assessment. The Upper Elementary team agreed that the data correlated with a lack of materials to navigate from concrete to conceptual understanding of algebra in the classroom environment. We began with the language we used within our weekly math lessons. We focused on vocabulary terms that would be beneficial when students needed to transfer information from previously learned concepts to the new concepts and lessons. The math textbooks brought real-world examples of problem-solving, which brought purpose to the students' work.

We implemented Hands-On Equations to teach students to navigate from concrete to abstract understanding of algebraic functions. We also started Problem Solving Friday each week which required students to engage their problem-solving skills, teamwork capabilities, and critical thinking skills. Problem Solving Friday allowed students to apply their understanding and present their learning. When the class debriefed the problem-solving process, students talked through moments of self-doubt and frustration. Those conversations focused on intentional and positive self-talk about their abilities to solve problems, forge through complex problems, and remind themselves and each other of the work they already mastered.

When we started our action research project, we observed that many students avoided math practice. Math work was not marked off on work plans, follow-up was incomplete, and math was touched the least during the work cycle. The Montessori philosophy instructs guides to reflect on themselves and the environment when something is not normalized. We focused on math interventions that created purposeful work with that in mind.

Our students told us they felt intimidated by algebra because it required strong math facts skills and familiarity with new vocabulary. Math facts practice became a priority for the entire classroom. Acquiring math fact automaticity allows the brain to direct energy toward the effort needed to apply numerous mathematical skills to higher-level thinking problems (Baker & Cuevas, 2018). Students focused on learning new vocabulary and started making connections to the similarities and meanings of words—for example, equal, equivalent, and equations. Once students felt comfortable with the various mathematical vocabulary words, it was easier for them to decipher what the problem asked.

Problem Solving Friday showed that our students have a strong sense of pride in their presentation of learning. Upper elementary students are in a plane of development where they thrive when learning can be social. The students' motivation and sense of purpose improved when they knew they would present at the end of each week. They asked questions, sought guidance, completed their daily practice, and worked towards mastery.

Hands-On Equations gave concrete representations of integers and variables in algebraic equations. It taught our students algebraic concepts such as balancing equations using inverse operations. Once we started the Hands-On Equations lessons, our students frequently asked for more lessons and used the materials in their follow-up when appropriate. We observed that student confidence improved each time they successfully found the value of a variable within the expression presented and asked for more complex equations to solve.

The findings point to our interventions leading to an overall improvement in students' motivation and engagement and improving their algebraic and critical thinking skills. We will continue to include Hands-on Equations lessons as a staple in our math curriculum. Students frequently used the work and watched lessons again when other students received it. Students challenged themselves to use newly learned techniques to solve previous problems. We will continue to schedule Problem Solving Friday each week with math groups. Students initiated this weekly work. Peer collaboration is beneficial to students in upper elementary. Studies show that middle-grade students learn better and more enthusiastically when collaborating (Daugherty, 2014). Mastering math vocabulary comprehension is essential, so students understand what a problem is asking. For example, "evaluate the expression" means to solve the problem.

Our research will change the dynamics of our classroom in many ways. Students will learn how to apply the lessons they have learned rather than simply memorize rules. We will continue to build student confidence through supporting new vocabulary to make language transfer manageable. We will include elements of real-world application to make their work purposeful. Students' interest in math will continue to increase along with their confidence in their abilities.

We initially thought that the root of the problem was the lack of concrete materials for algebra. After reading through our observations, we realized the problem involved more than a lack of concrete understanding. Some of our students had difficulties working with the unfamiliar and would refuse even to try the work. That was probably the most challenging issue to handle, and we are still working on it. Their perspective on failing might be our next topic of research. Our data collected throughout the study reflected steady improvement and increased engagement with math work.

We used formal assessments to help guide our instruction based on observing what students understood and what they needed more work with. The students used a solutions manual and some teacher guidance to check their work on the assessments. This practice aligned well with the Montessori philosophy of using a control of error to check for understanding. Checking their own work allowed the students to work through their mistakes and immediately see whether they understood the work and made simple mistakes or needed extra practice. Checking their own work also offered a judgment-free path to scoring their assessments. Students should be involved with the entire assessment process, from taking the test to checking their work and then analyzing their work to understand the positive benefits of formal assessments. Whether it was correct or incorrect, thinking through their process helps them guide their own learning.

We recommend creating an environment that protects students' vulnerability. For reasons unknown at this time, students were internally afraid to fail. Their vulnerability was a critical component that helped us track student progress. It helped us look closer at the students who needed extra support. All students acknowledged that it was a challenging part of their growth throughout the study. At times we would see students feel unsure of their work and lack confidence when working with their math groups. These times offered a circumstance to show students that "failing" was a learning opportunity. This helped us know when to push them a little further with challenging work and help them through a productive struggle or when they were at their frustration limit.

Students must receive consistent support and encouragement when they reach their frustration limits to keep them in the productive struggle. Overcoming these challenges increases students' persistence and endurance and improves their confidence. During lessons, work cycle, and Problem Solving Friday, we intentionally reminded students that they had all of the tools they needed to succeed with their current work. Direct instruction on past lessons and how they connected to the current concepts allowed a more seamless transfer of understanding from what they already knew to what they were learning.

Montessori frequently teaches that students will come by this knowledge on their own. However, we found that students needed discussions of previous learning added to their lessons. For example, the word equivalent is taught in Geometry to mean the same value but in a different form. We also use the term equivalent when discussing fractions and decimals. For example, the connection can be made that ½ is equivalent to 0.5 or five-tenths, so they look different but are the same amount. We recommend that educators find ways to circle back to previous lessons or have discussions with students about how they are building on a new concept from their background knowledge.

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Appendix A

Problem Solving Friday Self-Assessment Rubric

1	I completely understand this and feel like I could teach it.
2	I understand this but feel like I could use more practice with it.
3	I'm close to understanding this, but still need more help.
4	I don't understand this at all.